Aufgaben Wurzeln

1. Überprüfe durch Potenzieren, ob die Aussage stimmt. Falls nötig, berichtige den Radikanden.

a) $\sqrt{125}=5$ b) $\sqrt[3]{64}=8$ c) $\sqrt[5]{320000}=20$ d) $\sqrt[7]{0{,}0000128}=0{,}2$

2. Berechne.

a) $64^{\frac{5}{6}}$ b) $32^{\frac{4}{5}}$ c) $0{,}25^{-\frac{1}{2}}$ d) $0{,}008^{\frac{2}{3}}$ e) $\left(\frac{9}{16}\right)^{-1{,}5}$

3. Schreibe ohne Wurzel.

a) $\left(\sqrt{6}\cdot\sqrt{7}\right)^2$ b) $\left(\sqrt{2}\cdot\sqrt{5}\right)^{-4}$ c) $\left(\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{7}\right)^8$ d) $\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^4$

4. Berechne.

a) $2^{\frac{1}{2}}:2^{\frac{1}{4}}$ b) $6^{\frac{3}{4}}:6^{-\frac{1}{4}}$ c) $5^{-\frac{1}{2}}:5^{\frac{1}{4}}$ d) $\left(\frac{2}{3}\right)^{\frac{3}{5}}:\left(\frac{2}{3}\right)^{\frac{2}{5}}$

e) $2^{\frac{1}{2}}\cdot 8^{\frac{1}{2}}$ f) $3^{\frac{3}{4}}\cdot 27^{\frac{3}{4}}$ g) $3^{-\frac{1}{2}}\cdot 12^{-\frac{1}{2}}$ h) $2^{\frac{1}{2}}\cdot 128^{\frac{1}{2}}$

5. Vereinfache.

a) $\sqrt[3]{2}:\sqrt{2}$ b) $\sqrt[6]{5}:\sqrt[3]{5}$ c) $\sqrt[8]{2}:\sqrt[4]{2}$ d) $\sqrt[4]{4}:\sqrt[4]{4}$

e) $\sqrt[5]{7}:\sqrt[3]{7}$ f) $\sqrt[7]{x}:\sqrt[8]{x}$ g) $\sqrt[12]{a}:\sqrt[3]{a}$ h) $\sqrt{b}:\sqrt[3]{b}$

6. Beseitige die Wurzel im Nenner.

a) $\sqrt[3]{\frac{1}{2}}$ b) $\sqrt[4]{\frac{2}{3}}$ c) $\sqrt[5]{\frac{7}{5}}$ d) $\sqrt[6]{\frac{1}{125}}$

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Lösungen

1. Überprüfe durch Potenzieren, ob die Aussage stimmt. Falls nötig, berichtige den Radikanden.

a) $\sqrt{125}=5$

$\sqrt{125}=5 \quad | \ ^2$

$125 = 25$

$\sqrt{25}=5$

b) $\sqrt[3]{64}=8$

$\sqrt[3]{64}=8 \quad | \ ^3$

$64 = 512$

$\sqrt[3]{512}=8$

c) $\sqrt[5]{320000}=20$

$\sqrt[5]{320000}=20 \quad | \ ^5$

$320000 = 3200000$

$\sqrt[5]{3200000}=20$

d) $\sqrt[7]{0{,}0000128}=0{,}2$

$\sqrt[7]{0{,}0000128}=0{,}2 \quad | \ ^7$

$0{,}0000128 = 0{,}0000128$

$\sqrt[7]{0{,}0000128}=0{,}2$

2. Berechne.

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a) $64^{\frac{5}{6}}$

$64^{\frac{5}{6}}=(\sqrt[6]{64})^5=(\sqrt[6]{2^6})^5=((2^6)^{\frac{1}{6}})^5=(2^{6\cdot\frac{1}{6}})^5=(2^{\frac{6}{6}})^5=(2^{\frac{1}{1}})^5=(2^1)^5=2^5=32$

b) $32^{\frac{4}{5}}$

$32^{\frac{4}{5}}=(\sqrt[5]{32})^4=(\sqrt[5]{2^5})^4=((2^5)^{\frac{1}{5}})^4=(2^{5\cdot\frac{1}{5}})^4=(2^{\frac{5}{5}})^4=(2^{\frac{1}{1}})^4=(2^1)^4=2^4=16$

c) $0{,}25^{-\frac{1}{2}}$

$0{,}25^{-\frac{1}{2}}=(\frac{1}{4})^{-\frac{1}{2}}=(\frac{4}{1})^{\frac{1}{2}}=4^{\frac{1}{2}}=(2^2)^{\frac{1}{2}}=2^{2\cdot\frac{1}{2}}=2^{\frac{2}{2}}=2^{\frac{1}{1}}=2^1=2$

d) $0{,}008^{\frac{2}{3}}$

$0{,}008^{\frac{2}{3}}=(\frac{1}{125})^{\frac{2}{3}}=\frac{1}{125^{\frac{2}{3}}}=\frac{1}{(5^3)^{\frac{2}{3}}}=\frac{1}{5^{3\cdot\frac{2}{3}}}=\frac{1}{5^{\frac{6}{3}}}=\frac{1}{5^2}=\frac{1}{25}=0{,}04$

e) $(\frac{9}{16})^{-1{,}5}$

$(\frac{9}{16})^{-1{,}5}=(\frac{9}{16})^{-\frac{3}{2}}=(\frac{16}{9})^{\frac{3}{2}}=(\frac{4^2}{3^2})^{\frac{3}{2}}=\frac{4^{2\cdot\frac{3}{2}}}{3^{2\cdot\frac{3}{2}}}=\frac{4^{\frac{6}{2}}}{3^{\frac{6}{2}}}=\frac{4^3}{3^3}=\frac{64}{27}$

3. Schreibe ohne Wurzel.

a) $(\sqrt{6}\cdot\sqrt{7})^2$

$(\sqrt{6}\cdot\sqrt{7})^2=(\sqrt{6\cdot7})^2=(\sqrt{42})^2=(42^{\frac{1}{2}})^2=42^{\frac{1}{2}\cdot2}=42^{\frac{2}{2}}=42^{\frac{1}{1}}=42^1=42$

b) $(\sqrt{2}\cdot\sqrt{5})^{-4}$

$(\sqrt{2}\cdot\sqrt{5})^{-4}=(\sqrt{2\cdot5})^{-4}=(\sqrt{10})^{-4}=(10^{\frac{1}{2}})^{-4}=10^{\frac{1}{2}\cdot(-4)}=10^{\frac{-4}{2}}=10^{-2}=\frac{1}{10^2}=\frac{1}{100}=0{,}01$

c) $(\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{7})^8$

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$(\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{7})^8=(\sqrt{2\cdot5\cdot7})^8=(\sqrt{70})^8=(70^{\frac{1}{2}})^8=70^{\frac{1}{2}\cdot8}=70^{\frac{8}{2}}=70^4=24010000$

d) $(\frac{\sqrt{3}}{\sqrt{2}})^4$

$(\frac{\sqrt{3}}{\sqrt{2}})^4=(\sqrt{\frac{3}{2}})^4=(\frac{3}{2})^{\frac{1}{2}\cdot4}=(\frac{3}{2})^{\frac{4}{2}}=(\frac{3}{2})^2=\frac{3^2}{2^2}=\frac{9}{4}=2{,}25$

4. Berechne.

a) $2^{\frac{1}{2}}:2^{\frac{1}{4}}$

$2^{\frac{1}{2}}:2^{\frac{1}{4}}=2^{\frac{1}{2}-\frac{1}{4}}=2^{\frac{2}{4}-\frac{1}{4}}=2^{\frac{1}{4}}=\sqrt[4]{2^1}=\sqrt[4]{2}$

b) $6^{\frac{3}{4}}:6^{-\frac{1}{4}}$

$6^{\frac{3}{4}}:6^{-\frac{1}{4}}=6^{\frac{3}{4}-(-\frac{1}{4})}=6^{\frac{4}{4}}=6^{\frac{1}{1}}=6^1=6$

c) $5^{-\frac{1}{2}}:5^{\frac{1}{4}}$

$5^{-\frac{1}{2}}:5^{\frac{1}{4}}=5^{-\frac{1}{2}-\frac{1}{4}}=5^{-\frac{2}{4}-\frac{1}{4}}=5^{-\frac{3}{4}}=\frac{1}{5^{\frac{3}{4}}}=\frac{1}{\sqrt[4]{5^3}}=\frac{1}{\sqrt[4]{125}}$

d) $(\frac{2}{3})^{\frac{3}{5}}:(\frac{2}{3})^{\frac{2}{5}}$

$(\frac{2}{3})^{\frac{3}{5}}:(\frac{2}{3})^{\frac{2}{5}}=(\frac{2}{3})^{\frac{3}{5}-\frac{2}{5}}=(\frac{2}{3})^{\frac{1}{5}}=\sqrt[5]{\frac{2^1}{3^1}}=\sqrt[5]{\frac{2}{3}}$

e) $2^{\frac{1}{2}}\cdot8^{\frac{1}{2}}$

$2^{\frac{1}{2}}\cdot8^{\frac{1}{2}}=(2\cdot8)^{\frac{1}{2}}=16^{\frac{1}{2}}=(4^2)^{\frac{1}{2}}=4^{2\cdot\frac{1}{2}}=4^{\frac{2}{2}}=4^{\frac{1}{1}}=4^1=4$

f) $3^{\frac{3}{4}}\cdot27^{\frac{3}{4}}$

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$3^{\frac{3}{4}} \cdot 27^{\frac{3}{4}} = (3 \cdot 27)^{\frac{3}{4}} = 81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^{4 \cdot \frac{3}{4}} = 3^{\frac{12}{4}} = 3^3 = 27$

g) $3^{-\frac{1}{2}} \cdot 12^{-\frac{1}{2}}$

$3^{-\frac{1}{2}} \cdot 12^{-\frac{1}{2}} = (3 \cdot 12)^{-\frac{1}{2}} = 36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{(6^2)^{\frac{1}{2}}} = \frac{1}{6^{2 \cdot \frac{1}{2}}} = \frac{1}{6^{\frac{2}{2}}} = \frac{1}{6^{\frac{1}{1}}} = \frac{1}{6^1} = \frac{1}{6}$

h) $2^{\frac{1}{2}} \cdot 128^{\frac{1}{2}}$

$2^{\frac{1}{2}} \cdot 128^{\frac{1}{2}} = (2 \cdot 128)^{\frac{1}{2}} = 256^{\frac{1}{2}} = (16^2)^{\frac{1}{2}} = 16^{2 \cdot \frac{1}{2}} = 16^{\frac{2}{2}} = 16^{\frac{1}{1}} = 16^1 = 16$

5. Vereinfache.

a) $\sqrt[3]{2} : \sqrt{2}$

$\sqrt[3]{2} : \sqrt{2} = 2^{\frac{1}{3}} : 2^{\frac{1}{2}} = 2^{\frac{1}{3} - \frac{1}{2}} = 2^{\frac{2}{6} - \frac{3}{6}} = 2^{-\frac{1}{6}} = \frac{1}{2^{\frac{1}{6}}} = \frac{1}{\sqrt[6]{2^1}} = \frac{1}{\sqrt[6]{2}}$

b) $\sqrt[6]{5} : \sqrt[3]{5}$

$\sqrt[6]{5} : \sqrt[3]{5} = 5^{\frac{1}{6}} : 5^{\frac{1}{3}} = 5^{\frac{1}{6} - \frac{1}{3}} = 5^{\frac{1}{6} - \frac{2}{6}} = 5^{-\frac{1}{6}} = \frac{1}{5^{\frac{1}{6}}} = \frac{1}{\sqrt[6]{5^1}} = \frac{1}{\sqrt[6]{5}}$

c) $\sqrt[8]{2} : \sqrt[4]{2}$

$\sqrt[8]{2} : \sqrt[4]{2} = 2^{\frac{1}{8}} : 2^{\frac{1}{4}} = 2^{\frac{1}{8} - \frac{1}{4}} = 2^{\frac{1}{8} - \frac{2}{8}} = 2^{-\frac{1}{8}} = \frac{1}{2^{\frac{1}{8}}} = \frac{1}{\sqrt[8]{2^1}} = \frac{1}{\sqrt[8]{2}}$

d) $\sqrt[4]{4} : \sqrt[4]{4}$

$\sqrt[4]{4} : \sqrt[4]{4} = 4^{\frac{1}{4}} : 4^{\frac{1}{4}} = 4^{\frac{1}{4} - \frac{1}{4}} = 4^0 = 1$

e) $\sqrt[5]{7} : \sqrt[3]{7}$

$\sqrt[5]{7} : \sqrt[3]{7} = 7^{\frac{1}{5}} : 7^{\frac{1}{3}} = 7^{\frac{1}{5} - \frac{1}{3}} = 7^{\frac{3}{15} - \frac{5}{15}} = 7^{-\frac{2}{15}} = \frac{1}{\sqrt[15]{7^2}} = \frac{1}{\sqrt[15]{49}}$

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f) $\sqrt[7]{x} : \sqrt[8]{x}$

$\sqrt[7]{x} : \sqrt[8]{x} = x^{\frac{1}{7}} : x^{\frac{1}{8}} = x^{\frac{1}{7}-\frac{1}{8}} = x^{\frac{8}{56}-\frac{7}{56}} = x^{\frac{1}{56}} = \sqrt[56]{x^1} = \sqrt[56]{x}$

g) $\sqrt[12]{a} : \sqrt[3]{a}$

$\sqrt[12]{a} : \sqrt[3]{a} = a^{\frac{1}{12}} : a^{\frac{1}{3}} = a^{\frac{1}{12}-\frac{1}{3}} = a^{\frac{1}{12}-\frac{4}{12}} = a^{-\frac{3}{12}} = a^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{a^1}} = \frac{1}{\sqrt[4]{a}}$

h) $\sqrt{b} : \sqrt[3]{b}$

$\sqrt{b} : \sqrt[3]{b} = b^{\frac{1}{2}} : b^{\frac{1}{3}} = b^{\frac{1}{2}-\frac{1}{3}} = b^{\frac{3}{6}-\frac{2}{6}} = b^{\frac{1}{6}} = \sqrt[6]{b^1} = \sqrt[6]{b}$

6. Beseitige die Wurzel im Nenner.

a) $\sqrt[3]{\frac{1}{2}}$

$\sqrt[3]{\frac{1}{2}} = \sqrt[3]{\frac{1}{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{1 \cdot 4}}{\sqrt[3]{2 \cdot 4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2^3}} = \frac{\sqrt[3]{4}}{\left(2^3\right)^{\frac{1}{3}}} = \frac{\sqrt[3]{4}}{2^{3 \cdot \frac{1}{3}}} = \frac{\sqrt[3]{4}}{2^{\frac{3}{3}}} = \frac{\sqrt[3]{4}}{2^{\frac{1}{1}}} = \frac{\sqrt[3]{4}}{2^1} = \frac{\sqrt[3]{4}}{2}$

b) $\sqrt[4]{\frac{2}{3}}$

$\sqrt[4]{\frac{2}{3}} = \sqrt[4]{\frac{2}{3}} \cdot \frac{\sqrt[4]{27}}{\sqrt[4]{27}} = \frac{\sqrt[4]{2 \cdot 27}}{\sqrt[4]{3 \cdot 27}} = \frac{\sqrt[4]{54}}{\sqrt[4]{81}} = \frac{\sqrt[4]{54}}{\sqrt[4]{3^4}} = \frac{\sqrt[4]{54}}{\left(3^4\right)^{\frac{1}{4}}} = \frac{\sqrt[4]{54}}{3^{4 \cdot \frac{1}{4}}} = \frac{\sqrt[4]{54}}{3^{\frac{4}{4}}} = \frac{\sqrt[4]{54}}{3^{\frac{1}{1}}} = \frac{\sqrt[4]{54}}{3^1} = \frac{\sqrt[4]{54}}{3}$

c) $\sqrt[5]{\frac{7}{5}}$

$\sqrt[5]{\frac{7}{5}} = \sqrt[5]{\frac{7}{5}} \cdot \frac{\sqrt[5]{625}}{\sqrt[5]{625}} = \frac{\sqrt[5]{7 \cdot 625}}{\sqrt[5]{5 \cdot 625}} = \frac{\sqrt[5]{4375}}{\sqrt[5]{3125}} = \frac{\sqrt[5]{4375}}{\sqrt[5]{5^5}} = \frac{\sqrt[5]{4375}}{\left(5^5\right)^{\frac{1}{5}}} = \frac{\sqrt[5]{4375}}{5^{5 \cdot \frac{1}{5}}} = \frac{\sqrt[5]{4375}}{5^{\frac{5}{5}}} = \frac{\sqrt[5]{4375}}{5^{\frac{1}{1}}} = \frac{\sqrt[5]{4375}}{5^1} = \frac{\sqrt[5]{4375}}{5}$

d) $\sqrt[6]{\frac{1}{125}}$

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$$\sqrt[6]{\frac{1}{125}}=\sqrt[6]{\frac{1}{125}}\cdot\frac{\sqrt[6]{125}}{\sqrt[6]{125}}=\frac{\sqrt[6]{1\cdot 125}}{\sqrt[6]{125\cdot 125}}=\frac{\sqrt[6]{125}}{\sqrt[6]{15625}}=\frac{\sqrt[6]{125}}{\sqrt[6]{5^6}}=\frac{\sqrt[6]{125}}{(5^6)^{\frac{1}{6}}}=\frac{\sqrt[6]{125}}{5^{6\cdot\frac{1}{6}}}=\frac{\sqrt[6]{125}}{5^{\frac{6}{6}}}=\frac{\sqrt[6]{125}}{5^{\frac{1}{1}}}=\frac{\sqrt[6]{125}}{5^1}=\frac{\sqrt[6]{125}}{5}$$

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