

Aufgaben Wurzeln

1. Löse die Wurzel auf.

a) $\sqrt{\left(\frac{18}{89}\right)^2}$

b) $(-\sqrt{30})^2$

c) $(\sqrt{0,0064})^2$

d) $-(\sqrt{65})^2$

2. Bestimme die Definitionsmenge des Wurzelterms.

a) $\sqrt{x+7}$

b) $\sqrt{x-4}$

c) $\sqrt{8+4x}$

d) $\sqrt{10-a}$

e) $\sqrt{x^3+1}$

f) $\sqrt{x^2-4}$

g) $\sqrt{\frac{2}{x^2-4}}$

h) $\sqrt{x^2+4}$

3. Ermittle die Lösungsmenge.

a) $x^2=2500$

b) $x^2=-5$

c) $x^2=10000$

d) $x^2=5$

e) $x^2=6400$

f) $x^2=169$

g) $x^2=-196$

h) $x^2=7,29$

4. Berechne das Ergebnis.

a) $\sqrt{18} \cdot \sqrt{8}$

b) $\sqrt{54} \cdot \sqrt{6}$

c) $\sqrt{0,16 \cdot 49}$

d) $\sqrt{0,09 \cdot 256}$

5. Vereinfache.

a) $\sqrt{3x} \cdot \sqrt{3x^2y}$

b) $\sqrt{0,4x^2} \cdot \sqrt{0,9x^2}$

c) $\sqrt{\frac{x^2}{y}} : \sqrt{y}$

d) $\sqrt{x^3} : \sqrt{x}$

6. Vereinfache.

a) $4\sqrt{6}+5\sqrt{6}$

b) $9\sqrt{3}-3\sqrt{3}$

c) $-3\sqrt{11}+8\sqrt{11}$

d) $4\sqrt{2}-\sqrt{2}$

e) $5\sqrt{x}+2\sqrt{x}$

f) $8\sqrt{a}-4\sqrt{a}$

g) $\sqrt{25x}+\sqrt{x}$

h) $4\sqrt{a}-\sqrt{a}$

Lösungen

1. Löse die Wurzel auf.

a) $\sqrt{\left(\frac{18}{89}\right)^2}$

$$\sqrt{\left(\frac{18}{89}\right)^2} = \left[\left(\frac{18}{89}\right)^2\right]^{\frac{1}{2}} = \left(\frac{18}{89}\right)^{2 \cdot \frac{1}{2}} = \left(\frac{18}{89}\right)^{\frac{2}{2}} = \left(\frac{18}{89}\right)^1 = \left(\frac{18}{89}\right)^1 = \frac{18}{89}$$

b) $(-\sqrt{30})^2$

$$(-\sqrt{30})^2 = (-\sqrt{30}) \cdot (-\sqrt{30}) = (-1) \cdot (-1) \cdot (\sqrt{30}) \cdot (\sqrt{30}) = 1 \cdot [(\sqrt{30})^2]^2 = (30)^{\frac{1}{2} \cdot 2} = (30)^{\frac{2}{2}} = (30)^1 = 30^1 = 30$$

c) $(\sqrt{0,0064})^2$

$$(\sqrt{0,0064})^2 = [(0,0064)^{\frac{1}{2}}]^2 = (0,0064)^{\frac{1}{2} \cdot 2} = (0,0064)^{\frac{2}{2}} = (0,0064)^1 = 0,0064^1 = 0,0064$$

d) $-(\sqrt{65})^2$

$$-(\sqrt{65})^2 = -[(65)^{\frac{1}{2}}]^2 = -(65)^{\frac{1}{2} \cdot 2} = -(65)^{\frac{2}{2}} = -(65)^1 = -(65)^1 = -65$$

2. Bestimme die Definitionsmenge des Wurzelterms.

a) $\sqrt{x+7}$

$$x+7 \geq 0 \quad | -7$$

$$x \geq -7$$

$$D = \{x \mid x \geq -7\}$$

b) $\sqrt{x-4}$

$$x-4 \geq 0 \quad | +4$$

$$x \geq 4$$

$$D = \{x \mid x \geq 4\}$$

$$c) \sqrt{8+4x}$$

$$8+4x \geq 0 \quad | -8$$

$$4x \geq -8 \quad | :4$$

$$x \geq -2$$

$$D = \{x \mid x \geq -2\}$$

$$d) \sqrt{10-a}$$

$$10-a \geq 0 \quad | +a$$

$$a \leq 10$$

$$D = \{a \mid a \leq 10\}$$

$$e) \sqrt{x^3+1}$$

$$x^3+1 \geq 0 \quad | -1$$

$$x^3 \geq -1 \quad | \sqrt[3]{}$$

$$x \geq -1$$

$$D = \{x \mid x \geq -1\}$$

$$f) \sqrt{x^2-4}$$

$$x^2-4 \geq 0 \quad | +4$$

$$x^2 \geq 4 \quad | \sqrt{}$$

$$|x| \geq 2$$

$$\text{Für } x \geq 0$$

$$x \geq 2$$

$$\text{Für } x \leq 0$$

$$-x \geq 2 \quad | \cdot (-1)$$

$$x \leq -2$$

$$D = \{x \mid x \leq -2 \text{ oder } x \geq 2\}$$

$$g) \sqrt{\frac{2}{x^2 - 4}}$$

$$x^2 - 4 > 0 \quad | + 4$$

$$x^2 > 4 \quad | \sqrt{}$$

$$|x| > 2$$

Für $x > 0$

$$x > 2$$

Für $x < 0$

$$-x > 2 \quad | \cdot (-1)$$

$$x < -2$$

$$D = \{x \mid x < -2 \text{ oder } x > 2\}$$

$$h) \sqrt{x^2 + 4}$$

$$x^2 + 4 \geq 0 \quad | - 4$$

$$x^2 \geq -4 \quad | \sqrt{}$$

$x \geq$ nicht definiert in $\mathbb{R}$

$$D = \{x \mid x \in \mathbb{R}\}$$

3. Ermittle die Lösungsmenge.

$$a) x^2 = 2500$$

$$x^2 = 2500 \quad | \sqrt{}$$

$$x = \pm \sqrt{2500}$$

$$x = \pm 50$$

$$L = \{-50; 50\}$$

$$\text{b) } x^2 = -5$$

$$x^2 = -5 \quad | \sqrt{}$$

$x = \text{nicht definiert in } \mathbb{R}$

$$L = \emptyset$$

$$\text{c) } x^2 = 10000$$

$$x^2 = 10000 \quad | \sqrt{}$$

$$x = \pm \sqrt{10000}$$

$$x = \pm 100$$

$$L = \{-100; 100\}$$

$$\text{d) } x^2 = 5$$

$$x^2 = 5 \quad | \sqrt{}$$

$$x = \pm \sqrt{5}$$

$$L = \{-\sqrt{5}; \sqrt{5}\}$$

$$\text{e) } x^2 = 6400$$

$$x^2 = 6400 \quad | \sqrt{}$$

$$x = \pm \sqrt{6400}$$

$$x = \pm 80$$

$$L = \{-80; 80\}$$

$$\text{f) } x^2 = 169$$

$$x^2 = 169 \quad | \sqrt{}$$

$$x = \pm \sqrt{169}$$

$$x = \pm 13$$

$$L = \{-13; 13\}$$

$$g) x^2 = -196$$

$$x^2 = -196 \quad | \sqrt{}$$

$x = \text{nicht definiert in } \mathbb{R}$

$$L = \emptyset$$

$$h) x^2 = 7,29$$

$$x^2 = 7,29 \quad | \sqrt{}$$

$$x = \pm \sqrt{7,29}$$

$$x = \pm 2,7$$

$$L = \{-2,7; 2,7\}$$

4. Berechne das Ergebnis.

$$a) \sqrt{18} \cdot \sqrt{8}$$

$$\sqrt{18} \cdot \sqrt{8} = \sqrt{18 \cdot 8} = \sqrt{144} = \sqrt{12 \cdot 12} = \sqrt{12^2} = 12^{2^{\frac{1}{2}}} = 12^{2 \cdot \frac{1}{2}} = 12^{\frac{2}{2}} = 12^{\frac{1}{1}} = 12^1 = 12$$

$$b) \sqrt{54} \cdot \sqrt{6}$$

$$\sqrt{54} \cdot \sqrt{6} = \sqrt{54 \cdot 6} = \sqrt{324} = \sqrt{18 \cdot 18} = \sqrt{18^2} = 18^{2^{\frac{1}{2}}} = 18^{2 \cdot \frac{1}{2}} = 18^{\frac{2}{2}} = 18^{\frac{1}{1}} = 18^1 = 18$$

$$c) \sqrt{0,16 \cdot 49}$$

$$\sqrt{0,16 \cdot 49} = \sqrt{7,84} = \sqrt{2,8 \cdot 2,8} = \sqrt{2,8^2} = 2,8^{2^{\frac{1}{2}}} = 2,8^{2 \cdot \frac{1}{2}} = 2,8^{\frac{2}{2}} = 2,8^{\frac{1}{1}} = 2,8^1 = 2,8$$

$$d) \sqrt{0,09 \cdot 256}$$

$$\sqrt{0,09 \cdot 256} = \sqrt{23,04} = \sqrt{4,8 \cdot 4,8} = \sqrt{4,8^2} = 4,8^{2^{\frac{1}{2}}} = 4,8^{2 \cdot \frac{1}{2}} = 4,8^{\frac{2}{2}} = 4,8^{\frac{1}{1}} = 4,8^1 = 4,8$$

5. Vereinfache.

$$a) \sqrt{3x} \cdot \sqrt{3x^2y}$$

$$\sqrt{3x} \cdot \sqrt{3x^2y} = \sqrt{3x \cdot 3x^2y} = \sqrt{3^2 \cdot x^{2+1}y} = \sqrt{3^2 x^3 y} = (3^2 x^3 y)^{\frac{1}{2}} = 3^{2 \cdot \frac{1}{2}} x^{3 \cdot \frac{1}{2}} y^{1 \cdot \frac{1}{2}} = 3^{\frac{2}{2}} x^{\frac{3}{2}} y^{\frac{1}{2}} = 3x \sqrt{x} \sqrt{y}$$

$$b) \sqrt{0,4x^2} \cdot \sqrt{0,9x^2}$$

$$\sqrt{0,4x^2} \cdot \sqrt{0,9x^2} = \sqrt{0,4x^2 \cdot 0,9x^2} = \sqrt{0,36x^{2+2}} = \sqrt{0,6 \cdot 0,6x^4} = \sqrt{0,6^2 x^4} = (0,6^2 x^4)^{\frac{1}{2}} = 0,6^{2 \cdot \frac{1}{2}} x^{4 \cdot \frac{1}{2}} = 0,6^{\frac{2}{2}} x^{\frac{4}{2}} = 0,6x^2$$

$$c) \sqrt{\frac{x^2}{y}} : \sqrt{y}$$

$$\sqrt{\frac{x^2}{y}} : \sqrt{y} = \frac{\sqrt{\frac{x^2}{y}}}{\sqrt{y}} = \sqrt{\frac{\frac{x^2}{y}}{y}} = \sqrt{\frac{x^2}{y} : y} = \sqrt{\frac{x^2}{y} \cdot \frac{1}{y}} = \sqrt{\frac{x^2}{y^{1+1}}} = \sqrt{\frac{x^2}{y^2}} = \left(\frac{x^2}{y^2}\right)^{\frac{1}{2}} = \frac{x^{2 \cdot \frac{1}{2}}}{y^{2 \cdot \frac{1}{2}}} = \frac{x^{\frac{2}{2}}}{y^{\frac{2}{2}}} = \frac{x^{\frac{1}{1}}}{y^{\frac{1}{1}}} = \frac{x^1}{y^1} = \frac{x}{y}$$

$$d) \sqrt{x^3} : \sqrt{x}$$

$$\sqrt{x^3} : \sqrt{x} = \sqrt{\frac{x^3}{x}} = \sqrt{x^3 \cdot x^{-1}} = \sqrt{x^{3-1}} = \sqrt{x^2} = (x^2)^{\frac{1}{2}} = x^{2 \cdot \frac{1}{2}} = x^{\frac{2}{2}} = x^{\frac{1}{1}} = x^1 = x$$

6. Vereinfache.

$$a) 4\sqrt{6} + 5\sqrt{6}$$

$$4\sqrt{6} + 5\sqrt{6} = (4+5)\sqrt{6} = 9\sqrt{6}$$

$$b) 9\sqrt{3} - 3\sqrt{3}$$

$$9\sqrt{3} - 3\sqrt{3} = (9-3)\sqrt{3} = 6\sqrt{3}$$

$$c) -3\sqrt{11} + 8\sqrt{11}$$

$$-3\sqrt{11} + 8\sqrt{11} = (-3+8)\sqrt{11} = 5\sqrt{11}$$

$$d) 4\sqrt{2} - \sqrt{2}$$

$$4\sqrt{2} - \sqrt{2} = (4-1)\sqrt{2} = 3\sqrt{2}$$

$$e) 5\sqrt{x} + 2\sqrt{x}$$

$$5\sqrt{x}+2\sqrt{x}=(5+2)\sqrt{x}=7\sqrt{x}$$

$$\text{f) } 8\sqrt{a}-4\sqrt{a}$$

$$\text{f) } 8\sqrt{a}-4\sqrt{a}=(8-4)\sqrt{a}=4\sqrt{a}$$

$$\text{g) } \sqrt{25x}+\sqrt{x}$$

$$\sqrt{25x}+\sqrt{x}=\sqrt{25}\cdot\sqrt{x}+\sqrt{x}=5\sqrt{x}+\sqrt{x}=(5+1)\sqrt{x}=6\sqrt{x}$$

$$\text{h) } 4\sqrt{a}-\sqrt{a}$$

$$4\sqrt{a}-\sqrt{a}=(4-1)\sqrt{a}=3\sqrt{a}$$