

Aufgaben Wurzeln

1. Berechne die Potenzen.

a) $1^2; 2^2; 3^2; \dots; 23^2; 24^2; 25^2$

b) $1^3; 2^3; 3^3; \dots; 23^3; 24^3; 25^3$

2. Löse die Wurzel auf.

a) $\sqrt{25}$

b) $\sqrt{64}$

c) $\sqrt{169}$

d) $\sqrt{289}$

e) $\sqrt{121}$

f) $\sqrt[3]{8}$

g) $\sqrt[3]{27}$

h) $\sqrt[3]{125}$

i) $\sqrt[3]{343}$

j) $\sqrt[3]{1000}$

3. Ermittle das Ergebnis.

a) $(\sqrt{90})^2$

b) $(\sqrt{2})^2$

c) $(\sqrt{\frac{1}{175}})^2$

d) $\sqrt{55^2}$

e) $\sqrt{12,7^2}$

f) $(\sqrt[3]{5})^3$

g) $(\sqrt[3]{88})^3$

h) $(\sqrt[3]{\frac{1}{398}})^3$

i) $\sqrt[3]{18^3}$

j) $\sqrt[3]{48,3^3}$

4. Berechne das Ergebnis.

a) $\sqrt{2} \cdot \sqrt{32}$

b) $\sqrt{5} \cdot \sqrt{20}$

c) $\sqrt{12} \cdot \sqrt{3}$

d) $\sqrt{60} \cdot \sqrt{15}$

e) $\sqrt{16 \cdot 36}$

f) $\sqrt{25 \cdot 9}$

g) $\sqrt{144 \cdot 169}$

h) $\sqrt{225 \cdot 64}$

5. Vereinfache.

a) $\sqrt{x} \cdot \sqrt{x}$

b) $\sqrt{a^3} \cdot \sqrt{a}$

c) $\sqrt{ab^2} \cdot \sqrt{a}$

d) $\sqrt{20x} \cdot \sqrt{5x}$

6. Vereinfache.

a) $\sqrt{\frac{3x^2}{y^2}}$

b) $\frac{\sqrt{64x}}{\sqrt{y^2}}$

c) $\sqrt{\frac{100a}{b^2}}$

d) $\frac{\sqrt{xy^2}}{\sqrt{xy}}$

Lösungen

1. Berechne die Potenzen.

a) 1^2 ; 2^2 ; 3^2 ; ... ; 23^2 ; 24^2 ; 25^2

$$1^2 = 1; 2^2 = 4; 3^2 = 9; 4^2 = 16; 5^2 = 25; 6^2 = 36; 7^2 = 49; 8^2 = 64; 9^2 = 81; 10^2 = 100; 11^2 = 121; \\ 12^2 = 144; 13^2 = 169; 14^2 = 196; 15^2 = 225; 16^2 = 256; 17^2 = 289; 18^2 = 324; 19^2 = 361; 20^2 = 400; 21^2 = 441; 22^2 = 484; 23^2 = 529; 24^2 = 576; 25^2 = 625$$

b) 1^3 ; 2^3 ; 3^3 ; ... ; 23^3 ; 24^3 ; 25^3

$$1^3 = 1; 2^3 = 8; 3^3 = 27; 4^3 = 64; 5^3 = 125; 6^3 = 216; 7^3 = 343; 8^3 = 512; 9^3 = 729; 10^3 = 1000; \\ 11^3 = 1331; 12^3 = 1728; 13^3 = 2197; 14^3 = 2744; 15^3 = 3375; 16^3 = 4096; 17^3 = 4913; 18^3 = 5832; 19^3 = 6859; 20^3 = 8000; 21^3 = 9261; 22^3 = 10648; 23^3 = 12167; 24^3 = 13824; 25^3 = 15625$$

2. Löse die Wurzel auf.

a) $\sqrt{25}$

$$\sqrt{25} = \sqrt{5 \cdot 5} = \sqrt{5^2} = (5^2)^{\frac{1}{2}} = 5^{2 \cdot \frac{1}{2}} = 5^{\frac{2}{2}} = 5^1 = 5^1 = 5$$

b) $\sqrt{64}$

$$\sqrt{64} = \sqrt{8 \cdot 8} = \sqrt{8^2} = (8^2)^{\frac{1}{2}} = 8^{2 \cdot \frac{1}{2}} = 8^{\frac{2}{2}} = 8^1 = 8^1 = 8$$

c) $\sqrt{169}$

$$\sqrt{169} = \sqrt{13 \cdot 13} = \sqrt{13^2} = (13^2)^{\frac{1}{2}} = 13^{2 \cdot \frac{1}{2}} = 13^{\frac{2}{2}} = 13^1 = 13^1 = 13$$

d) $\sqrt{289}$

$$\sqrt{289} = \sqrt{17 \cdot 17} = \sqrt{17^2} = (17^2)^{\frac{1}{2}} = 17^{2 \cdot \frac{1}{2}} = 17^{\frac{2}{2}} = 17^1 = 17^1 = 17$$

e) $\sqrt{121}$

$$\sqrt{121} = \sqrt{11 \cdot 11} = \sqrt{11^2} = (11^2)^{\frac{1}{2}} = 11^{2 \cdot \frac{1}{2}} = 11^{\frac{2}{2}} = 11^{\frac{1}{1}} = 11^1 = 11$$

f) $\sqrt[3]{8}$

$$\sqrt[3]{8} = \sqrt{2 \cdot 2 \cdot 2} = \sqrt{2^3} = (2^3)^{\frac{1}{3}} = 2^{3 \cdot \frac{1}{3}} = 2^{\frac{3}{3}} = 2^{\frac{1}{1}} = 2^1 = 2$$

g) $\sqrt[3]{27}$

$$\sqrt[3]{27} = \sqrt{3 \cdot 3 \cdot 3} = \sqrt{3^3} = (3^3)^{\frac{1}{3}} = 3^{3 \cdot \frac{1}{3}} = 3^{\frac{3}{3}} = 3^{\frac{1}{1}} = 3^1 = 3$$

h) $\sqrt[3]{125}$

$$\sqrt[3]{125} = \sqrt{5 \cdot 5 \cdot 5} = \sqrt{5^3} = (5^3)^{\frac{1}{3}} = 5^{3 \cdot \frac{1}{3}} = 5^{\frac{3}{3}} = 5^{\frac{1}{1}} = 5^1 = 5$$

i) $\sqrt[3]{343}$

$$\sqrt[3]{343} = \sqrt{7 \cdot 7 \cdot 7} = \sqrt{7^3} = (7^3)^{\frac{1}{3}} = 7^{3 \cdot \frac{1}{3}} = 7^{\frac{3}{3}} = 7^{\frac{1}{1}} = 7^1 = 7$$

j) $\sqrt[3]{1000}$

$$\sqrt[3]{1000} = \sqrt{10 \cdot 10 \cdot 10} = \sqrt{10^3} = (10^3)^{\frac{1}{3}} = 10^{3 \cdot \frac{1}{3}} = 10^{\frac{3}{3}} = 10^{\frac{1}{1}} = 10^1 = 10$$

3. Ermittle das Ergebnis.

a) $(\sqrt{90})^2$

$$(\sqrt{90})^2 = (90^{\frac{1}{2}})^2 = 90^{\frac{1}{2} \cdot 2} = 90^{\frac{2}{2}} = 90^{\frac{1}{1}} = 90^1 = 90$$

b) $(\sqrt{2})^2$

$$(\sqrt{2})^2 = (2^{\frac{1}{2}})^2 = 2^{\frac{1}{2} \cdot 2} = 2^{\frac{2}{2}} = 2^{\frac{1}{1}} = 2^1 = 2$$

c) $(\sqrt{\frac{1}{175}})^2$

$$(\sqrt{\frac{1}{175}})^2 = \left[\left(\frac{1}{175} \right)^{\frac{1}{2}} \right]^2 = \left(\frac{1}{175} \right)^{\frac{1}{2} \cdot 2} = \left(\frac{1}{175} \right)^{\frac{2}{2}} = \left(\frac{1}{175} \right)^{\frac{1}{1}} = \left(\frac{1}{175} \right)^1 = \frac{1}{175}$$

d) $\sqrt{55^2}$

$$(\sqrt{55})^2 = (55^{\frac{1}{2}})^2 = 55^{\frac{1}{2} \cdot 2} = 55^{\frac{2}{2}} = 55^{\frac{1}{1}} = 55^1 = 55$$

e) $\sqrt{12,7^2}$

$$\sqrt{12,7^2} = 12,7^{\frac{1}{2}} = 12,7^{2 \cdot \frac{1}{2}} = 12,7^{\frac{2}{2}} = 12,7^{\frac{1}{1}} = 12,7^1 = 12,7$$

f) $(\sqrt[3]{5})^3$

$$(\sqrt[3]{5})^3 = (5^{\frac{1}{3}})^3 = 5^{\frac{1}{3} \cdot 3} = 5^{\frac{3}{3}} = 5^{\frac{1}{1}} = 5^1 = 5$$

g) $(\sqrt[3]{88})^3$

$$(\sqrt[3]{88})^3 = (88^{\frac{1}{3}})^3 = 88^{\frac{1}{3} \cdot 3} = 88^{\frac{3}{3}} = 88^{\frac{1}{1}} = 88^1 = 88$$

h) $(\sqrt[3]{\frac{1}{398}})^3$

$$(\sqrt[3]{\frac{1}{398}})^3 = \left[\left(\frac{1}{398}\right)^{\frac{1}{3}}\right]^3 = \left(\frac{1}{398}\right)^{\frac{1}{3} \cdot 3} = \left(\frac{1}{398}\right)^{\frac{3}{3}} = \left(\frac{1}{398}\right)^{\frac{1}{1}} = \left(\frac{1}{398}\right)^1 = \frac{1}{398}$$

i) $\sqrt[3]{18^3}$

$$(\sqrt[3]{18})^3 = (18^{\frac{1}{3}})^3 = 18^{\frac{1}{3} \cdot 3} = 18^{\frac{3}{3}} = 18^{\frac{1}{1}} = 18^1 = 18$$

j) $\sqrt[3]{48,3^3}$

$$(\sqrt[3]{48,3})^3 = (48,3^{\frac{1}{3}})^3 = 48,3^{\frac{1}{3} \cdot 3} = 48,3^{\frac{3}{3}} = 48,3^{\frac{1}{1}} = 48,3^1 = 48,3$$

4. Berechne das Ergebnis.

a) $\sqrt{2} \cdot \sqrt{32}$

$$\sqrt{2} \cdot \sqrt{32} = \sqrt{2 \cdot 32} = \sqrt{64} = \sqrt{8 \cdot 8} = (8^2)^{\frac{1}{2}} = 8^{2 \cdot \frac{1}{2}} = 8^{\frac{2}{2}} = 8^{\frac{1}{1}} = 8^1 = 8$$

$$\text{b) } \sqrt{5} \cdot \sqrt{20}$$

$$\sqrt{5} \cdot \sqrt{20} = \sqrt{5 \cdot 20} = \sqrt{100} = \sqrt{10 \cdot 10} = (10^2)^{\frac{1}{2}} = 10^{2 \cdot \frac{1}{2}} = 10^{\frac{2}{2}} = 10^{\frac{1}{1}} = 10^1 = 10$$

$$\text{c) } \sqrt{12} \cdot \sqrt{3}$$

$$\sqrt{5} \cdot \sqrt{20} = \sqrt{5 \cdot 20} = \sqrt{100} = \sqrt{10 \cdot 10} = (10^2)^{\frac{1}{2}} = 10^{2 \cdot \frac{1}{2}} = 10^{\frac{2}{2}} = 10^{\frac{1}{1}} = 10^1 = 10$$

$$\text{d) } \sqrt{60} \cdot \sqrt{15}$$

$$\sqrt{60} \cdot \sqrt{15} = \sqrt{60 \cdot 15} = \sqrt{900} = \sqrt{30 \cdot 30} = (30^2)^{\frac{1}{2}} = 30^{2 \cdot \frac{1}{2}} = 30^{\frac{2}{2}} = 30^{\frac{1}{1}} = 30^1 = 30$$

$$\text{e) } \sqrt{16 \cdot 36}$$

$$\sqrt{16 \cdot 36} = \sqrt{576} = \sqrt{24 \cdot 24} = (24^2)^{\frac{1}{2}} = 24^{2 \cdot \frac{1}{2}} = 24^{\frac{2}{2}} = 24^{\frac{1}{1}} = 24^1 = 24$$

$$\text{f) } \sqrt{25 \cdot 9}$$

$$\sqrt{25 \cdot 9} = \sqrt{225} = \sqrt{15 \cdot 15} = (15^2)^{\frac{1}{2}} = 15^{2 \cdot \frac{1}{2}} = 15^{\frac{2}{2}} = 15^{\frac{1}{1}} = 15^1 = 15$$

$$\text{g) } \sqrt{144 \cdot 169}$$

$$\sqrt{144 \cdot 169} = \sqrt{24336} = \sqrt{156 \cdot 156} = (156^2)^{\frac{1}{2}} = 156^{2 \cdot \frac{1}{2}} = 156^{\frac{2}{2}} = 156^{\frac{1}{1}} = 156^1 = 156$$

$$\text{h) } \sqrt{225 \cdot 64}$$

$$\sqrt{225 \cdot 64} = \sqrt{14400} = \sqrt{120 \cdot 120} = (120^2)^{\frac{1}{2}} = 120^{2 \cdot \frac{1}{2}} = 120^{\frac{2}{2}} = 120^{\frac{1}{1}} = 120^1 = 120$$

5. Vereinfache.

$$\text{a) } \sqrt{x} \cdot \sqrt{x}$$

$$\sqrt{x} \cdot \sqrt{x} = \sqrt{x \cdot x} = x^{2 \cdot \frac{1}{2}} = x^{2 \cdot \frac{1}{2}} = x^{\frac{2}{2}} = x^{\frac{1}{1}} = x^1 = x$$

$$\text{b) } \sqrt{a^3} \cdot \sqrt{a}$$

$$\sqrt{a^3} \cdot \sqrt{a} = \sqrt{a^3 \cdot a^1} = \sqrt{a^{3+1}} = \sqrt{a^4} = a^{4 \cdot \frac{1}{2}} = a^{4 \cdot \frac{1}{2}} = a^{\frac{4}{2}} = a^{\frac{2}{1}} = a^2$$

$$c) \sqrt{ab^2} \cdot \sqrt{a}$$

$$\sqrt{ab^2} \cdot \sqrt{a} = \sqrt{a^1 b^2 \cdot a^1} = \sqrt{a^{1+1} b^2} = \sqrt{a^2 b^2} = (ab)^{2^{\frac{1}{2}}} = (ab)^{2 \cdot \frac{1}{2}} = (ab)^{\frac{2}{2}} = (ab)^{\frac{1}{1}} = (ab)^1 = ab$$

$$d) \sqrt{20x} \cdot \sqrt{5x}$$

$$\sqrt{20x} \cdot \sqrt{5x} = \sqrt{20x^1 \cdot 5x^1} = \sqrt{100x^{1+1}} = \sqrt{10 \cdot 10x^2} = \sqrt{(10x)^2} = (10x)^{2 \cdot \frac{1}{2}} = (10x)^{\frac{2}{2}} = (10x)^{\frac{1}{1}} = (10x)^1 = 10x$$

6. Vereinfache.

$$a) \sqrt{\frac{3x^2}{y^2}}$$

$$\sqrt{\frac{3x^2}{y^2}} = \left(\frac{3x^2}{y^2}\right)^{\frac{1}{2}} = \frac{3^{\frac{1}{2}} x^{2 \cdot \frac{1}{2}}}{y^{2 \cdot \frac{1}{2}}} = \frac{3^{\frac{1}{2}} x^{2 \cdot \frac{1}{2}}}{y^{2 \cdot \frac{1}{2}}} = \frac{3^{\frac{1}{2}} x^{\frac{2}{2}}}{y^{\frac{2}{2}}} = \frac{3^{\frac{1}{2}} x^1}{y^1} = \frac{3^{\frac{1}{2}} x^1}{y^1} = \frac{\sqrt{3} x}{y}$$

$$b) \frac{\sqrt{64x}}{\sqrt{y^2}}$$

$$\frac{\sqrt{64x}}{\sqrt{y^2}} = \frac{\sqrt{8 \cdot 8x}}{y^{2^{\frac{1}{2}}}} = \frac{\sqrt{8^2 x}}{y^{2 \cdot \frac{1}{2}}} = \frac{(8^2 x)^{\frac{1}{2}}}{y^{\frac{2}{2}}} = \frac{8^{2 \cdot \frac{1}{2}} x^{\frac{1}{2}}}{y^1} = \frac{8^{\frac{2}{2}} \sqrt{x}}{y^1} = \frac{8^{\frac{1}{1}} \sqrt{x}}{y} = \frac{8^1 \sqrt{x}}{y} = \frac{8\sqrt{x}}{y}$$

$$c) \sqrt{\frac{100a}{b^2}}$$

$$\sqrt{\frac{100a}{b^2}} = \frac{\sqrt{10 \cdot 10a}}{b^{2^{\frac{1}{2}}}} = \frac{\sqrt{10^2 a}}{b^{2 \cdot \frac{1}{2}}} = \frac{(10^2 a)^{\frac{1}{2}}}{b^{\frac{2}{2}}} = \frac{10^{2 \cdot \frac{1}{2}} a^{\frac{1}{2}}}{b^1} = \frac{10^{\frac{2}{2}} \sqrt{a}}{b^1} = \frac{10^{\frac{1}{1}} \sqrt{a}}{b} = \frac{10^1 \sqrt{a}}{b} = \frac{10\sqrt{a}}{b}$$

$$d) \frac{\sqrt{xy^2}}{\sqrt{xy}}$$

$$\frac{\sqrt{xy^2}}{\sqrt{xy}} = \sqrt{\frac{xy^2}{xy}} = \sqrt{x^1 y^2 x^{-1} y^{-1}} = \sqrt{x^{1-1} y^{2-1}} = \sqrt{x^0 y^1} = \sqrt{1 \cdot y} = \sqrt{y}$$