

Aufgaben zu quadratischen Gleichungen

1. Ermittle die Lösungsmenge der quadratischen Gleichung.

a) $x^2 + 9 = 0$

b) $3x^2 - 12 = 0$

c) $-5y^2 + 0,2 = 0$

d) $4y^2 - 9 = 0$

e) $4y^2 + 9 = 0$

f) $-0,5x^2 + 8x = 0$

g) $-0,5x^2 + 8 = 0$

h) $9a^2 - 4 = 0$

i) $4a - 9a^2 = 60$

j) $x^2 + 6x + 9 = 0$

2. Ermittle mittels der p-q-Formel die Lösungsmenge.

a) $3x^2 - 15x + 12 = 0$

b) $2x^2 + 14x - 25,5 = 0$

c) $5x^2 + 25x - 10 = 0$

d) $9x^2 + 66x + 137 = 0$

3. Ermittle die Lösungsmenge. Klammere hierbei nicht aus!

a) $(2x - 5)(2x^2 - x - 10) = 0$

b) $(25x^2 + 20x + 4)(10x + 4) = 0$

c) $(4x + 9)(x^2 + 4x + 9) = 0$

d) $(7y + 2)(4y^2 - 28y + 49) = 0$

4. Ermittle jeweils auf die schnellste Art und Weise die Lösungsmenge.

a) $9x^2 + 16x = 0$

b) $10x^2 - 24x + 18 = 0$

c) $5x^2 = 12x$

d) $12x^2 - 3 = 0$

Lösungen

1. Ermittle die Lösungsmenge der quadratischen Gleichung.

a) $x^2 + 9 = 0$

$$x^2 + 9 = 0 \quad | -9$$

$$x^2 = -9 \quad | \sqrt{}$$

$x = \text{nicht definiert in } \mathbb{Q}$

$$L = \emptyset$$

b) $3x^2 - 12 = 0$

$$3x^2 - 12 = 0 \quad | +12$$

$$3x^2 = 12 \quad | :3$$

$$x^2 = 4 \quad | \sqrt{}$$

$$x = \pm 2$$

$$L = \{-2; 2\}$$

c) $-5y^2 + 0,2 = 0$

$$-5y^2 + 0,2 = 0 \quad | -0,2$$

$$-5y^2 = -0,2 \quad | :(-5)$$

$$y^2 = 0,04 \quad | \sqrt{}$$

$$y = \pm 0,2$$

$$L = \{-0,2; 0,2\}$$

d) $4y^2 - 9 = 0$

$$4y^2 - 9 = 0 \quad | + 9$$

$$4y^2 = 9 \quad | : 4$$

$$y^2 = 2,25 \quad | \sqrt{}$$

$$y = \pm 1,5$$

$$L = \{-1,5; 1,5\}$$

e) $4y^2 + 9 = 0$

$$4y^2 + 9 = 0 \quad | - 9$$

$$4y^2 = -9 \quad | : 4$$

$$y^2 = -2,25 \quad | \sqrt{}$$

$$y = \text{nicht definiert in } \mathbb{Q}$$

$$L = \emptyset$$

f) $-0,5x^2 + 8x = 0$

$$-0,5x^2 + 8x = 0$$

$$x \cdot (-0,5x + 8) = 0$$

$$x = 0$$

$$-0,5x + 8 = 0 \quad | - 8$$

$$-0,5x = -8 \quad | : (-0,5)$$

$$x = 16$$

$$L = \{0; 16\}$$

g) $-0,5x^2 + 8 = 0$

$$-0,5x^2 + 8 = 0 \quad | - 8$$

$$-0,5x^2 = -8 \quad | : (-0,5)$$

$$x^2 = 16 \quad | \sqrt{}$$

$$x = \pm 4$$

$$L = \{-4; 4\}$$

$$\text{h) } 9a^2 - 4 = 0$$

$$9a^2 - 4 = 0 \quad | + 4$$

$$9a^2 = 4 \quad | : 9$$

$$a^2 = \frac{4}{9}$$

$$a^2 = \frac{4}{9} \quad | \sqrt{}$$

$$a = \pm \frac{2}{3}$$

$$L = \left\{-\frac{2}{3}; \frac{2}{3}\right\}$$

$$\text{i) } 4a - 9a^2 = 60$$

$$4a - 9a^2 = 60 \quad | - 60$$

$$-9a^2 + 4a - 60 = 0 \quad | : (-9)$$

$$a^2 - \frac{4}{9}a + \frac{60}{9}$$

$$a^2 - \frac{4}{9}a + \frac{20}{3}$$

$$a_{1,2} = \frac{\frac{4}{9}}{2} \pm \sqrt{\left(\frac{\frac{4}{9}}{2}\right)^2 - \frac{20}{3}}$$

$$a_{1,2} = \frac{2}{9} \pm \sqrt{\left(\frac{2}{9}\right)^2 - \frac{20}{3}}$$

$$a_{1,2} = \frac{2}{9} \pm \sqrt{\frac{4}{81} - \frac{20}{3}}$$

$$a_{1,2} = \frac{2}{9} \pm \sqrt{-\frac{536}{81}}$$

$$a_{1,2} = \frac{2}{9} \pm \text{nicht definiert in } \mathbb{Q}$$

$$L = \emptyset$$

j) $x^2 + 6x + 9 = 0$

$$x_{1,2} = -\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - 9}$$

$$x_{1,2} = -3 \pm \sqrt{(3)^2 - 9}$$

$$x_{1,2} = -3 \pm \sqrt{9 - 9}$$

$$x_{1,2} = -3 \pm \sqrt{0}$$

$$x_{1,2} = -3 \pm 0$$

$$x_{1,2} = -3$$

$$L = \{-3\}$$

2. Ermittle mittels der p-q-Formel die Lösungsmenge.

a) $3x^2 - 15x + 12 = 0$

$$3x^2 - 15x + 12 = 0 \quad | : 3$$

$$x^2 - 5x + 4 = 0$$

$$x_{1,2} = \frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4}$$

$$x_{1,2} = 2,5 \pm \sqrt{(2,5)^2 - 4}$$

$$x_{1,2} = 2,5 \pm \sqrt{6,25 - 4}$$

$$x_{1,2} = 2,5 \pm \sqrt{2,25}$$

$$x_{1,2} = 2,5 \pm 1,5$$

$$x_1 = 2,5 - 1,5$$

$$x_1 = 1$$

$$x_2 = 2,5 + 1,5$$

$$x_2 = 4$$

$$L = \{1; 4\}$$

$$\text{b) } 2x^2 + 14x - 25,5 = 0$$

$$2x^2 + 14x - 25,5 = 0 \quad | : 2$$

$$x^2 + 7x - 12,75 = 0$$

$$x_{1,2} = -\frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 + 12,75}$$

$$x_{1,2} = -3,5 \pm \sqrt{(3,5)^2 + 12,75}$$

$$x_{1,2} = -3,5 \pm \sqrt{12,25 + 12,75}$$

$$x_{1,2} = -3,5 \pm \sqrt{25}$$

$$x_{1,2} = -3,5 \pm 5$$

$$x_1 = -3,5 - 5$$

$$x_1 = -8,5$$

$$x_2 = -3,5 + 5$$

$$x_2 = 1,5$$

$$L = \{-8,5; 1,5\}$$

$$c) 5x^2 + 25x - 10 = 0$$

$$5x^2 + 25x - 10 = 0 \quad | : 5$$

$$x^2 + 5x - 2 = 0$$

$$x_{1,2} = -\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 + 2}$$

$$x_{1,2} = -2,5 \pm \sqrt{(2,5)^2 + 2}$$

$$x_{1,2} = -2,5 \pm \sqrt{6,25 + 2}$$

$$x_{1,2} = -2,5 \pm \sqrt{8,25}$$

$$x_1 = -2,5 - \sqrt{8,25} \approx -5,37$$

$$x_2 = -2,5 + \sqrt{8,25} \approx 0,37$$

$$L = \{-5,37; 0,37\}$$

$$d) 9x^2 + 66x + 137 = 0$$

$$9x^2 + 66x + 137 = 0 \quad | : 9$$

$$x^2 + \frac{66}{9}x + \frac{137}{9} = 0$$

$$x^2 + \frac{22}{3}x + \frac{137}{9} = 0$$

$$x_{1,2} = -\frac{\frac{22}{3}}{2} \pm \sqrt{\left(\frac{\frac{22}{3}}{2}\right)^2 - \frac{137}{9}}$$

$$x_{1,2} = -\frac{11}{3} \pm \sqrt{\left(\frac{11}{3}\right)^2 - \frac{137}{9}}$$

$$x_{1,2} = -\frac{11}{3} \pm \sqrt{\frac{121}{9} - \frac{137}{9}}$$

$$x_{1,2} = -\frac{11}{3} \pm \sqrt{-\frac{16}{9}}$$

$$x_{1,2} = -\frac{11}{3} \pm \text{nicht definiert in } \mathbb{Q}$$

$$L = \emptyset$$

3. Ermittle die Lösungsmenge. Klammere hierbei nicht aus!

$$\text{a) } (2x - 5)(2x^2 - x - 10) = 0$$

$$(2x - 5)(2x^2 - x - 10) = 0$$

$$2x - 5 = 0 \quad | + 5$$

$$2x = 5 \quad | : 2$$

$$x = 2,5$$

$$2x^2 - x - 10 = 0 \quad | : 2$$

$$x^2 - 0,5x - 5 = 0$$

$$x_{1,2} = \frac{0,5}{2} \pm \sqrt{\left(\frac{0,5}{2}\right)^2 + 5}$$

$$x_{1,2} = 0,25 \pm \sqrt{(0,25)^2 + 5}$$

$$x_{1,2} = 0,25 \pm \sqrt{0,0625 + 5}$$

$$x_{1,2} = 0,25 \pm \sqrt{5,0625}$$

$$x_{1,2} = 0,25 \pm 2,25$$

$$x_1 = 0,25 - 2,25 = -2$$

$$x_2 = 0,25 + 2,25 = 2,5$$

$$L = \{-2; 2,5\}$$

$$\text{b) } (25x^2 + 20x + 4)(10x + 4) = 0$$

$$(25x^2 + 20x + 4)(10x + 4) = 0$$

$$10x + 4 = 0 \quad | -4$$

$$10x = -4 \quad | : 10$$

$$x = -0,4$$

$$25x^2 + 20x + 4 = 0 \quad | : 25$$

$$x^2 + 0,8x + 0,16 = 0$$

$$x_{1,2} = -\frac{0,8}{2} \pm \sqrt{\left(\frac{0,8}{2}\right)^2 - 0,16}$$

$$x_{1,2} = -0,4 \pm \sqrt{(0,4)^2 - 0,16}$$

$$x_{1,2} = -0,4 \pm \sqrt{0,16 - 0,16}$$

$$x_{1,2} = -0,4 \pm \sqrt{0}$$

$$x_{1,2} = -0,4 \pm 0$$

$$x_{1,2} = -0,4$$

$$L = \{-0,4\}$$

$$\text{c) } (4x + 9)(x^2 + 4x + 9) = 0$$

$$(4x + 9)(x^2 + 4x + 9) = 0$$

$$4x + 9 = 0 \quad | -9$$

$$4x = -9 \quad | : 4$$

$$x = -2,25$$

$$x^2 + 4x + 9 = 0$$

$$x_{1,2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 9}$$

$$x_{1,2} = -2 \pm \sqrt{(2)^2 - 9}$$

$$x_{1,2} = -2 \pm \sqrt{4 - 9}$$

$$x_{1,2} = -2 \pm \sqrt{-5}$$

$$x_{1,2} = -2 \pm \text{nicht definiert in } \mathbb{Q}$$

$$L = \{-2,25\}$$

$$\text{d) } (7y + 2)(4y^2 - 28y + 49) = 0$$

$$(7y + 2)(4y^2 - 28y + 49) = 0$$

$$7y + 2 = 0 \quad | - 2$$

$$7y = -2 \quad | : 7$$

$$y = -\frac{2}{7}$$

$$4y^2 - 28y + 49 \quad | : 4$$

$$y^2 - 7y + 12,25 = 0$$

$$y_{1,2} = \frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 - 12,25}$$

$$y_{1,2} = \frac{7}{2} \pm \sqrt{\frac{49}{4} - 12,25}$$

$$y_{1,2} = \frac{7}{2} \pm \sqrt{0}$$

$$y_{1,2} = \frac{7}{2} \pm 0$$

$$y_{1,2} = \frac{7}{2}$$

$$L = \left\{ -\frac{2}{7}; \frac{7}{2} \right\}$$

4. Ermittle jeweils auf die schnellste Art und Weise die Lösungsmenge.

a) $9x^2 + 16x = 0$

$$9x^2 + 16x = 0$$

$$x \cdot (9x + 16) = 0$$

$$x = 0$$

$$9x + 16 = 0 \quad | -16$$

$$9x = -16 \quad | :9$$

$$x = -\frac{16}{9}$$

$$L = \left\{ -\frac{16}{9}; 0 \right\}$$

b) $10x^2 - 24x + 18 = 0$

$$10x^2 - 24x + 18 = 0 \quad | :10$$

$$x^2 - 2,4x + 1,8 = 0$$

$$x_{1,2} = \frac{2,4}{2} \pm \sqrt{\left(\frac{2,4}{2}\right)^2 - 1,8}$$

$$x_{1,2} = 1,2 \pm \sqrt{(1,2)^2 - 1,8}$$

$$x_{1,2} = 1,2 \pm \sqrt{1,44 - 1,8}$$

$$x_{1,2} = 1,2 \pm \sqrt{-0,36}$$

$$x_{1,2} = 1,2 \pm \text{nicht definiert in } \mathbb{Q}$$

$$L = \emptyset$$

$$\text{c) } 5x^2 = 12x$$

$$5x^2 = 12x \quad | - 12x$$

$$5x^2 - 12x = 0$$

$$x \cdot (5x - 12) = 0$$

$$x = 0$$

$$5x - 12 = \quad | + 12$$

$$5x = 12 \quad | : 5$$

$$x = 2,4$$

$$L = \{0; 2,4\}$$

$$\text{d) } 12x^2 - 3 = 0$$

$$12x^2 - 3 \quad | + 3$$

$$12x^2 = 3 \quad | : 12$$

$$x^2=\frac{3}{12}$$

$$x^2=\frac{1}{4}$$

$$x^2=\frac{1}{4} \quad | \sqrt{}$$

$$x=\pm \frac{1}{2}$$

$$L=\{-\frac{1}{2};\frac{1}{2}\}$$