

Aufgaben zu binomischen Formeln

1. Ermittle die Lösungsmenge.

a) $(x - 4)^2 = (x + 5)^2$

b) $(x + 7)(x - 7) = (x + 8)^2 - 1$

c) $-(x + 9)^2 + (x - 11)^2 = 0$

d) $(2x + 5)(8x - 3) - 5 = (4x - 1)^2$

e) $\frac{5}{36} - (x - \frac{1}{2})(x + \frac{1}{2}) + (x + \frac{1}{3})^2 = 0$

f) $(3x - 8)(3x + 8) - (x + 1,2)(9x - 5) = -87$

g) $(1,5x - 3,2)^2 - 38,08 = (1,5x + 2,4)^2$

h) $-(\frac{x}{2} - 2)(\frac{x}{8} + 5) + (\frac{x}{4} + 1)(\frac{x}{4} - 1) + 9 = 0$

2. Bestimme die Lösungsmenge.

a) $(x - 2) \cdot (x + 7) = (x + 1)^2$

b) $(6 - 2a)^2 = (2a - 4)^2 - 36$

c) $(2x - 7)^2 + 9 = 4x^2 - 9x + 1$

d) $6y + (4 - y)^2 = -(24 - y^2)$

e) $(y - 5) \cdot (y + 3) - y = (y - 3)^2$

f) $(1 + 2x)^2 - 30x - 14 = (2x - 15)^2$

g) $(1 + 3z)^2 + 8z + 1 = (2 + 3z)^2$

h) $(3 - x)^2 + 5x = (x + 2) \cdot (3 + x)$

i) $(t + 3)(t - 3) + 2 = (t - 7)^2$

j) $(3 + 2t)(1 - 8t) + 20 = (1 - 4t)(1 + 4t)$

3. Fülle die Unterstriche innerhalb der Gleichung aus.

a) $(4a + _)^2 = _ + _ + 64$

b) $(_ - 6)^2 = _ - 96y + _$

c) $(_ + _)^2 = 9a^2 + 24a + _$

d) $(_ + _)^2 = _ + 720a + 400$

e) $(_ - _)^2 = 144a^2 - 96a + _$

f) $(_ - _) \cdot (_ + _) = 144x^2 - 36y^2$

g) $(_ - 3a) \cdot (_ + _) = -_ + 4x^2$

h) $(12 + _) \cdot (_ - _) = _ - 400q^2$

4. Faktorisiere und ziehe hierfür die binomischen Formeln heran.

a)

$$x^2 + 2xy + y^2$$

$$x^2 + 2x \cdot 7 + 49$$

$$a^2 - 2ab + b^2$$

$$r^2 - 2rs + s^2$$

b)

$$x^2 + 20x + 100$$

$$a^2 - 20a + 100$$

$$x^2 - 2x + 1$$

$$a^2 + 14a + 49$$

c)

$$25 + a^2 - 10a$$

$$36 + x^2 + 12x$$

$$100 + b^2 - 20b$$

$$169 + x^2 - 26x$$

d)

$$a^2 - b^2$$

$$u^2 - 121v^2$$

$$1 - 16r^2$$

$$s^2 - 625$$

Lösungen

1. Ermittle die Lösungsmenge.

a) $(x - 4)^2 = (x + 5)^2$

$$(x - 4)^2 = (x + 5)^2$$

$$(x)^2 - 2 \cdot x \cdot 4 + (4)^2 = (x)^2 + 2 \cdot x \cdot 5 + (5)^2$$

$$x^2 - 8x + 16 = x^2 + 10x + 25 \quad | -x^2$$

$$-8x + 16 = 10x + 25 \quad | -16$$

$$-8x = 10x + 9 \quad | -10x$$

$$-18x = 9 \quad | :(-18)$$

$$x = -0,5$$

$$L = \{-0,5\}$$

b) $(x + 7)(x - 7) = (x + 8)^2 - 1$

$$(x + 7)(x - 7) = (x + 8)^2 - 1$$

$$x \cdot x + 7 \cdot (-7) = (x)^2 + 2 \cdot x \cdot 8 + (8)^2 - 1$$

$$x^2 - 49 = x^2 + 16x + 64 - 1$$

$$x^2 - 49 = x^2 + 16x + 63 \quad | -x^2$$

$$-49 = 16x + 63 \quad | -63$$

$$-112 = 16x \quad | :16$$

$$-7 = x$$

$$L = \{-7\}$$

c) $-(x + 9)^2 + (x - 11)^2 = 0$

$$-(x + 9)^2 + (x - 11)^2 = 0$$

$$-((x)^2 + 2 \cdot x \cdot 9 + (9)^2) + (x)^2 - 2 \cdot x \cdot 11 + (11)^2 = 0$$

$$-(x^2 + 18x + 81) + x^2 - 22x + 121 = 0$$

$$-x^2 - 18x - 81 + x^2 - 22x + 121 = 0$$

$$-40x + 40 = 0 \quad | -40$$

$$-40x = -40 \quad | : (-40)$$

$$x = 1$$

$$L = \{1\}$$

$$d) (2x + 5)(8x - 3) - 5 = (4x - 1)^2$$

$$(2x + 5)(8x - 3) - 5 = (4x - 1)^2$$

$$2x \cdot 8x + 2x \cdot (-3) + 5 \cdot 8x + 5 \cdot (-3) - 5 = (4x)^2 - 2 \cdot 4x \cdot 1 + (1)^2$$

$$16x^2 - 6x + 40x - 15 - 5 = 16x^2 - 8x + 1$$

$$16x^2 + 34x - 20 = 16x^2 - 8x + 1 \quad | -16x^2$$

$$\dots 34x - 20 = -8x + 1 \quad | +8x$$

$$42x - 20 = 1 \quad | +20$$

$$42x = 21 \quad | :42$$

$$x = 0,5$$

$$L = \{0,5\}$$

$$e) \frac{5}{36} - (x - \frac{1}{2})(x + \frac{1}{2}) + (x + \frac{1}{3})^2 = 0$$

$$\frac{5}{36} - (x - \frac{1}{2})(x + \frac{1}{2}) + (x + \frac{1}{3})^2 = 0$$

$$\frac{5}{36} - (x \cdot x + (-\frac{1}{2}) \cdot \frac{1}{2}) + (x)^2 + 2 \cdot x \cdot \frac{1}{3} + (\frac{1}{3})^2 = 0$$

$$\frac{5}{36} - (x^2 - \frac{1}{4}) + x^2 + \frac{2}{3}x + \frac{1}{9} = 0$$

$$\frac{5}{36} - x^2 + \frac{1}{4} + x^2 + \frac{2}{3}x + \frac{1}{9} = 0$$

$$\frac{1}{2} + \frac{2}{3}x = 0 \quad | -\frac{1}{2}$$

$$\frac{2}{3}x = -\frac{1}{2} \quad | : \frac{2}{3}$$

$$x = -\frac{3}{4}$$

$$L = \left\{-\frac{3}{4}\right\}$$

$$f) (3x - 8)(3x + 8) - (x + 1,2)(9x - 5) = -87$$

$$(3x - 8)(3x + 8) - (x + 1,2)(9x - 5) = -87$$

$$3x \cdot 3x + (-8) \cdot 8 - (x \cdot 9x + x \cdot (-5) + 1,2 \cdot 9x + 1,2 \cdot (-5)) = -87$$

$$9x^2 - 64 - (9x^2 - 5x + 10,8x - 6) = -87$$

$$9x^2 - 64 - 9x^2 + 5x - 10,8x + 6 = -87$$

$$-58 - 5,8x = -87 \quad | + 58$$

$$-5,8x = -29 \quad | : (-5,8)$$

$$x = 5$$

$$L = \{5\}$$

$$g) (1,5x - 3,2)^2 - 38,08 = (1,5x + 2,4)^2$$

$$(1,5x - 3,2)^2 - 38,08 = (1,5x + 2,4)^2$$

$$(1,5x)^2 - 2 \cdot 1,5x \cdot 3,2 + (3,2)^2 - 38,08 = (1,5x)^2 + 2 \cdot 1,5x \cdot 2,4 + (2,4)^2$$

$$2,25x^2 - 9,6x + 10,24 - 38,08 = 2,25x^2 + 7,2x + 5,76$$

$$2,25x^2 - 9,6x - 27,84 = 2,25x^2 + 7,2x + 5,76 \quad | - 2,25x^2$$

$$-9,6x - 27,84 = 7,2x + 5,76 \quad | + 27,84$$

$$-9,6x = 7,2x + 33,6 \quad | - 7,2x$$

$$-16,8x = 33,6 \quad | : (-16,8)$$

$$x = -2$$

$$L = \{-2\}$$

$$h) -\left(\frac{x}{2}-2\right)\left(\frac{x}{8}+5\right)+\left(\frac{x}{4}+1\right)\left(\frac{x}{4}-1\right)+9=0$$

$$-\left(\frac{x}{2}-2\right)\left(\frac{x}{8}+5\right)+\left(\frac{x}{4}+1\right)\left(\frac{x}{4}-1\right)+9=0$$

$$-\left(\frac{x}{2} \cdot \frac{x}{8} + \frac{x}{2} \cdot 5 + (-2) \cdot \frac{x}{8} + (-2) \cdot 5\right) + \left(\frac{x}{4}\right) \cdot \left(\frac{x}{4}\right) + 1 \cdot (-1) + 9 = 0$$

$$-\left(\frac{x^2}{16} + \frac{5x}{2} - \frac{2x}{8} - 10\right) + \left(\frac{x^2}{16}\right) - 1 + 9 = 0$$

$$-\left(\frac{x^2}{16} + \frac{5x}{2} - \frac{2x}{8} - 10\right) + \frac{x^2}{16} - 1 + 9 = 0$$

$$-\left(\frac{x^2}{16} + \frac{9x}{4} - 10\right) + \frac{x^2}{16} + 8 = 0$$

$$-\frac{x^2}{16} - \frac{9x}{4} + 10 + \frac{x^2}{16} + 8 = 0$$

$$-\frac{9x}{4} + 18 = 0 \quad | -18$$

$$-\frac{9x}{4} = -18 \quad | : \left(-\frac{9}{4}\right)$$

$$x = 8$$

$$L = \{8\}$$

2. Bestimme die Lösungsmenge.

$$a) (x-2) \cdot (x+7) = (x+1)^2$$

$$(x-2) \cdot (x+7) = (x+1)^2$$

$$x \cdot x + x \cdot 7 + (-2) \cdot x + (-2) \cdot 7 = (x)^2 + 2 \cdot x \cdot 1 + (1)^2$$

$$x^2 + 7x - 2x - 14 = x^2 + 2x + 1$$

$$x^2 + 5x - 14 = x^2 + 2x + 1 \quad | -x^2$$

$$5x - 14 = 2x + 1 \quad | -2x$$

$$3x - 14 = 1 \quad | +14$$

$$3x = 15 \quad | :3$$

$$x = 5$$

$$L = \{5\}$$

$$b) (6 - 2a)^2 = (2a - 4)^2 - 36$$

$$(6 - 2a)^2 = (2a - 4)^2 - 36$$

$$(6)^2 - 2 \cdot 6 \cdot 2a + (2a)^2 = (2a)^2 - 2 \cdot 2a \cdot 4 + (4)^2 - 36$$

$$36 - 24a + 4a^2 = 4a^2 - 16a + 16 - 36$$

$$36 - 24a + 4a^2 = 4a^2 - 16a - 20 \quad | - 4a^2$$

$$36 - 24a = -16a - 20 \quad | + 24a$$

$$36 = 8a - 20 \quad | + 20$$

$$56 = 8a \quad | : 8$$

$$a = 7$$

$$L = \{7\}$$

$$c) (2x - 7)^2 + 9 = 4x^2 - 9x + 1$$

$$(2x - 7)^2 + 9 = 4x^2 - 9x + 1$$

$$(2x)^2 - 2 \cdot 2x \cdot 7 + (7)^2 + 9 = 4x^2 - 9x + 1$$

$$4x^2 - 28x + 49 + 9 = 4x^2 - 9x + 1$$

$$4x^2 - 28x + 49 + 9 = 4x^2 - 9x + 1$$

$$4x^2 - 28x + 58 = 4x^2 - 9x + 1 \quad | - 4x^2$$

$$-28x + 58 = -9x + 1 \quad | + 28x$$

$$58 = 19x + 1 \quad | - 1$$

$$57 = 19x \quad | : 19$$

$$3 = x$$

$$L = \{3\}$$

$$d) 6y + (4 - y)^2 = -(24 - y^2)$$

$$6y + (4 - y)^2 = -(24 - y^2)$$

$$6y + (4)^2 - 2 \cdot 4 \cdot y + (y)^2 = -(24 - y^2)$$

$$6y + 16 - 8y + y^2 = -24 + y^2$$

$$-2y + 16 + y^2 = -24 + y^2 \quad | -y^2$$

$$-2y + 16 = -24 \quad | -16$$

$$-2y = -40 \quad | :(-2)$$

$$y = 20$$

$$L = \{20\}$$

$$e) (y - 5) \cdot (y + 3) - y = (y - 3)^2$$

$$(y - 5) \cdot (y + 3) - y = (y - 3)^2$$

$$y \cdot y + y \cdot 3 + (-5) \cdot y + (-5) \cdot 3 - y = (y)^2 - 2 \cdot y \cdot 3 + (3)^2$$

$$y^2 + 3y - 5y - 15 - y = y^2 - 6y + 9$$

$$y^2 - 3y - 15 = y^2 - 6y + 9 \quad | -y^2$$

$$-3y - 15 = -6y + 9 \quad | +6y$$

$$3y - 15 = 9 \quad | +15$$

$$3y = 24 \quad | :3$$

$$y = 8$$

$$L = \{8\}$$

$$f) (1 + 2x)^2 - 30x - 14 = (2x - 15)^2$$

$$(1 + 2x)^2 - 30x - 14 = (2x - 15)^2$$

$$(1)^2 + 2 \cdot 1 \cdot 2x + (2x)^2 - 30x - 14 = (2x)^2 - 2 \cdot 2x \cdot 15 + (15)^2$$

$$1 + 4x + 4x^2 - 30x - 14 = 4x^2 - 60x + 225$$

$$-13 - 26x + 4x^2 = 4x^2 - 60x + 225 \quad | -4x^2$$

$$-13 - 26x = -60x + 225 \quad | +60x$$

$$-13 + 34x = 225 \quad | +13$$

$$34x = 238 \quad | : 34$$

$$x = 7$$

$$L = \{7\}$$

$$g) (1 + 3z)^2 + 8z + 1 = (2 + 3z)^2$$

$$(1 + 3z)^2 + 8z + 1 = (2 + 3z)^2$$

$$(1)^2 + 2 \cdot 1 \cdot 3z + (3z)^2 + 8z + 1 = (2)^2 + 2 \cdot 2 \cdot 3z + (3z)^2$$

$$1 + 6z + 9z^2 + 8z + 1 = 4 + 12z + 9z^2$$

$$2 + 14z + 9z^2 = 4 + 12z + 9z^2 \quad | - 9z^2$$

$$2 + 14z = 4 + 12z \quad | - 12z$$

$$2 + 2z = 4 \quad | - 2$$

$$2z = 2 \quad | : 2$$

$$z = 1$$

$$L = \{1\}$$

$$h) (3 - x)^2 + 5x = (x + 2) \cdot (3 + x)$$

$$(3 - x)^2 + 5x = (x + 2) \cdot (3 + x)$$

$$(3)^2 - 2 \cdot 3 \cdot x + (x)^2 + 5x = x \cdot 3 + x \cdot x + 2 \cdot 3 + 2x$$

$$9 - 6x + x^2 + 5x = 3x + x^2 + 6 + 2x$$

$$9 - x + x^2 = 5x + x^2 + 6 \quad | - x^2$$

$$9 - x = 5x + 6 \quad | + x$$

$$9 = 6x + 6 \quad | - 6$$

$$3 = 6x \quad | : 6$$

$$0,5 = x$$

$$L = \{0,5\}$$

$$i) (t + 3)(t - 3) + 2 = (t - 7)^2$$

$$(t + 3)(t - 3) + 2 = (t - 7)^2$$

$$t \cdot t - 3 \cdot 3 + 2 = (t)^2 - 2 \cdot t \cdot 7 + (7)^2$$

$$t^2 - 9 + 2 = t^2 - 14t + 49$$

$$t^2 - 7 = t^2 - 14t + 49 \quad | -t^2$$

$$-7 = -14t + 49 \quad | -49$$

$$-56 = -14t \quad | :(-14)$$

$$4 = t$$

$$L = \{4\}$$

$$j) (3 + 2t)(1 - 8t) + 20 = (1 - 4t)(1 + 4t)$$

$$(3 + 2t)(1 - 8t) + 20 = (1 - 4t)(1 + 4t)$$

$$3 \cdot 1 + 3 \cdot (-8t) + 2t \cdot 1 + 2t \cdot (-8t) + 20 = 1 \cdot 1 - 4t \cdot 4t$$

$$4 - 24t + 2t - 16t^2 + 20 = 2 - 16t^2$$

$$24 - 22t - 16t^2 = 2 - 16t^2 \quad | +16t^2$$

$$24 - 22t = 2 \quad | -24$$

$$-22t = -22 \quad | :(-22)$$

$$t = 1$$

$$L = \{1\}$$

3. Fülle die Unterstriche innerhalb der Gleichung aus.

$$a) (4a + _)^2 = _ + _ + 64$$

$$(4a + 8)^2 = 16a^2 + 64a + 64$$

$$b) (_ - 6)^2 = _ - 96y + _$$

$$(8y - 6)^2 = 64y^2 - 96y + 36$$

$$c) (_ + _)^2 = 9a^2 + 24a + _$$

$$(3a + 4)^2 = 9a^2 + 24a + 16$$

$$d) (_ + _)^2 = _ + 720a + 400$$

$$(18a + 20)^2 = 324a^2 + 720a + 400$$

$$e) (_ - _)^2 = 144a^2 - 96a + _$$

$$(12a - 4)^2 = 144a^2 - 96a + 16$$

$$f) (_ - _) \cdot (_ + _) = 144x^2 - 36y^2$$

$$(12x - 6y) \cdot (12x + 6y) = 144x^2 - 36y^2$$

$$g) (_ - 3a) \cdot (_ + _) = -_ + 4x^2$$

$$(2x - 3a) \cdot (2x + 3a) = -9a^2 + 4x^2$$

$$h) (12 + _) \cdot (_ - _) = _ - 400q^2$$

$$(12 + 20q) \cdot (12 - 20q) = 144 - 400q^2$$

4. Faktorisiere und ziehe hierfür die binomischen Formeln heran.

a)

$$x^2 + 2xy + y^2 = (x + y) \cdot (x + y) = (x + y)^2$$

$$x^2 + 2x \cdot 7 + 49 = (x + 7) \cdot (x + 7) = (x + 7)^2$$

$$a^2 - 2ab + b^2 = (a - b) \cdot (a - b) = (a - b)^2$$

$$r^2 - 2rs + s^2 = (r - s) \cdot (r - s) = (r - s)^2$$

b)

$$x^2 + 20x + 100 = (x + 10) \cdot (x + 10) = (x + 10)^2$$

$$a^2 - 20a + 100 = (a - 10) \cdot (a - 10) = (a - 10)^2$$

$$x^2 - 2x + 1 = (x - 1) \cdot (x - 1) = (x - 1)^2$$

$$a^2 + 14a + 49 = (a + 7) \cdot (a + 7) = (a + 7)^2$$

c)

$$25 + a^2 - 10a = (5 - a) \cdot (5 - a) = (5 - a)^2$$

$$36 + x^2 + 12x = (6 + x) \cdot (6 + x) = (6 + x)^2$$

$$100 + b^2 - 20b = (10 - b) \cdot (10 - b) = (10 - b)^2$$

$$169 + x^2 - 26x = (13 - x) \cdot (13 - x) = (13 - x)^2$$

d)

$$a^2 - b^2 = (a + b) \cdot (a - b)$$

$$u^2 - 121v^2 = (u + 11v) \cdot (u - 11v)$$

$$1 - 16r^2 = (1 + 4r) \cdot (1 - 4r)$$

$$s^2 - 625 = (s + 25) \cdot (s - 25)$$