

Aufgaben Wurzeln

1. Forme um zur Potenz.

a) $\sqrt{5}$

b) $\sqrt[4]{6}$

c) $\sqrt[9]{7}$

d) $\sqrt[3]{3^2}$

e) $\sqrt[5]{4^4}$

f) $\sqrt[8]{7^3}$

g) $\frac{4}{\sqrt{64}}$

h) $\sqrt[3]{\frac{1}{3^2}}$

i) $\sqrt{7^{-3}}$

j) $\sqrt[4]{\frac{1}{5^2}}$

2. Forme um in die Wurzelschreibweise.

a) $x^{\frac{1}{3}}$

b) $a^{\frac{1}{2}}$

c) $y^{\frac{2}{3}}$

d) $x^{\frac{3}{4}}$

$$e) x^{-\frac{2}{5}}$$

$$f) x^{-\frac{3}{4}}$$

$$g) a^{0,5}$$

$$h) x^{7,2}$$

$$i) a^{0,8}$$

$$j) m^{-5,1}$$

$$k) m^{-2,4}$$

$$l) a^{-0,55}$$

3. Forme zu Potenz mit rationalem Exponenten um.

$$a) \sqrt{x^4 + y^4}$$

$$b) \sqrt[5]{(a \cdot b)^2}$$

$$c) \sqrt{x \cdot y}$$

$$d) \frac{1}{\sqrt[5]{a}}$$

$$e) \frac{6}{\sqrt{a^n}}$$

$$f) \frac{-3}{\sqrt[n]{x-y}}$$

$$g) \frac{5}{\sqrt[3n]{a+b}}$$

$$h) \frac{-2}{\sqrt[n+my]{x^2+1}}$$

4. Wende ein Wurzelgesetz an, um die Potenzen richtig aufzulösen.

a) $4^{\frac{4}{3}} \cdot 4^{\frac{2}{3}}$

b) $3^{\frac{1}{4}} \cdot 3^{\frac{3}{4}}$

c) $2^{\frac{1}{6}} \cdot 2^{-\frac{1}{3}}$

d) $7^{-\frac{3}{5}} \cdot 7^{-\frac{8}{5}}$

e) $\frac{5^{\frac{3}{4}}}{3} \cdot \frac{5^{-\frac{1}{4}}}{3}$

f) $\frac{1^{-\frac{2}{3}}}{2} \cdot \frac{1^{-\frac{1}{2}}}{2}$

Lösungen

1. Forme um zur Potenz.

a) $\sqrt{5}$

$$\sqrt{5} = 5^{\frac{1}{2}}$$

b) $\sqrt[4]{6}$

$$\sqrt[4]{6} = 6^{\frac{1}{4}}$$

c) $\sqrt[9]{7}$

$$\sqrt[9]{7} = 7^{\frac{1}{9}}$$

d) $\sqrt[3]{3^2}$

$$\sqrt[3]{3^2} = 3^{\frac{2}{3}}$$

e) $\sqrt[5]{4^4}$

$$\sqrt[5]{4^4} = 4^{\frac{4}{5}}$$

f) $\sqrt[8]{7^3}$

$$\sqrt[8]{7^3} = 7^{\frac{3}{8}}$$

g) $\frac{4}{\sqrt{64}}$

$$\frac{4}{\sqrt{64}} = \frac{4}{64^{\frac{1}{2}}} = 4 \cdot 64^{-\frac{1}{2}}$$

$$\text{h) } \sqrt[3]{\frac{1}{3^2}}$$

$$\sqrt[3]{\frac{1}{3^2}} = \left(\frac{1}{3^2}\right)^{\frac{1}{3}} = (3^{-2})^{\frac{1}{3}} = 3^{(-2) \cdot \frac{1}{3}} = 3^{-\frac{1}{3}}$$

$$\text{i) } \sqrt{7^{-3}}$$

$$\sqrt{7^{-3}} = (7^{-3})^{\frac{1}{2}} = 7^{-3 \cdot \frac{1}{2}} = 7^{-\frac{3}{2}}$$

$$\text{j) } \sqrt[4]{\frac{1}{5^2}}$$

$$\sqrt[4]{\frac{1}{5^2}} = \sqrt[4]{5^{-2}} = (5^{-2})^{\frac{1}{4}} = 5^{-2 \cdot \frac{1}{4}} = 5^{-\frac{2}{4}} = 5^{-\frac{1}{2}}$$

2. Forme um in die Wurzelschreibweise.

$$\text{a) } x^{\frac{1}{3}}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$\text{b) } a^{\frac{1}{2}}$$

$$a^{\frac{1}{2}} = \sqrt[2]{a} = \sqrt{a}$$

$$\text{c) } y^{\frac{2}{3}}$$

$$y^{\frac{2}{3}} = \sqrt[3]{y^2}$$

$$d) x^{\frac{3}{4}}$$

$$x^{\frac{3}{4}} = \sqrt[4]{x^3}$$

$$e) x^{-\frac{2}{5}}$$

$$x^{-\frac{2}{5}} = \frac{1}{x^{\frac{2}{5}}} = \frac{1}{\sqrt[5]{x^2}}$$

$$f) x^{-\frac{3}{4}}$$

$$x^{-\frac{3}{4}} = \frac{1}{x^{\frac{3}{4}}} = \sqrt[4]{x^3}$$

$$g) a^{0,5}$$

$$a^{0,5} = a^{\frac{5}{10}} = a^{\frac{1}{2}} = \sqrt[2]{a} = \sqrt{a}$$

$$h) x^{7,2}$$

$$x^{7,2} = x^{\frac{72}{10}} = x^{\frac{36}{5}} = \sqrt[5]{x^{36}}$$

$$i) a^{0,8}$$

$$a^{0,8} = a^{\frac{8}{10}} = a^{\frac{4}{5}} = \sqrt[5]{a^4}$$

$$j) m^{-5,1}$$

$$m^{-5,1} = \frac{1}{m^{5,1}} = \frac{1}{m^{\frac{51}{10}}} = \frac{1}{\sqrt[10]{m^{51}}}$$

k) $m^{-2,4}$

$$m^{-2,4} = \frac{1}{m^{2,4}} = \frac{1}{m^{\frac{24}{10}}} = \frac{1}{m^{\frac{12}{5}}} = \frac{1}{\sqrt[5]{m^{12}}}$$

l) $a^{-0,55}$

$$a^{-0,55} = \frac{1}{a^{0,55}} = \frac{1}{a^{\frac{55}{100}}} = \frac{1}{a^{\frac{11}{20}}} = \frac{1}{\sqrt[20]{a^{11}}}$$

3. Forme zu Potenz mit rationalem Exponenten um.

a) $\sqrt{x^4 + y^4}$

$$\sqrt{x^4 + y^4} = (x^4 + y^4)^{\frac{1}{2}}$$

b) $\sqrt[5]{(a \cdot b)^2}$

$$\sqrt[5]{(a \cdot b)^2} = (a \cdot b)^{\frac{2}{5}}$$

c) $\sqrt{x \cdot y}$

$$\sqrt{x \cdot y} = (x \cdot y)^{\frac{1}{2}}$$

d) $\frac{1}{\sqrt[5]{a}}$

$$\frac{1}{\sqrt[5]{a}} = \frac{1}{a^{\frac{1}{5}}} = a^{-\frac{1}{5}}$$

$$\text{e) } \frac{6}{\sqrt[n]{a^n}}$$

$$\frac{6}{\sqrt[n]{a^n}} = \frac{6}{a^{\frac{n}{n}}} = 6 \cdot a^{-\frac{n}{n}} = 6 a^{-1}$$

$$\text{f) } \frac{-3}{\sqrt[n]{x-y}}$$

$$\frac{-3}{\sqrt[n]{x-y}} = \frac{-3}{(x-y)^{\frac{1}{n}}} = -3 \cdot (x-y)^{-\frac{1}{n}} = -3(x-y)^{-\frac{1}{n}}$$

$$\text{g) } \frac{5}{\sqrt[3n]{a+b}}$$

$$\frac{5}{\sqrt[3n]{a+b}} = \frac{5}{(a+b)^{\frac{1}{3n}}} = 5(a+b)^{-\frac{1}{3n}}$$

$$\text{h) } \frac{-2}{\sqrt[n+m]{x^2+1}}$$

$$\frac{-2}{\sqrt[n+m]{x^2+1}} = \frac{-2}{(x^2+1)^{\frac{1}{n+m}}} = -2 \cdot (x^2+1)^{-\frac{1}{n+m}} = -2(x^2+1)^{-\frac{1}{n+m}}$$

4. Wende ein Wurzelgesetz an, um die Potenzen richtig aufzulösen.

$$\text{a) } 4^{\frac{4}{3}} \cdot 4^{\frac{2}{3}}$$

$$4^{\frac{4}{3}} \cdot 4^{\frac{2}{3}} = 4^{\frac{4}{3} + \frac{2}{3}} = 4^{\frac{6}{3}} = 4^2 = 4 \cdot 4 = 16$$

$$\text{b) } 3^{\frac{1}{4}} \cdot 3^{\frac{3}{4}}$$

$$3^{\frac{1}{4}} \cdot 3^{\frac{3}{4}} = 3^{\frac{1}{4} + \frac{3}{4}} = 3^{\frac{4}{4}} = 3^1 = 3$$

$$\text{c) } 2^{\frac{1}{6}} \cdot 2^{-\frac{1}{3}}$$

$$2^{\frac{1}{6}} \cdot 2^{-\frac{1}{3}} = 2^{\frac{1}{6} - \frac{1}{3}} = 2^{\frac{1}{6} - \frac{2}{6}} = 2^{-\frac{1}{6}} = \frac{1}{2^{\frac{1}{6}}} = \frac{1}{\sqrt[6]{2}}$$

$$\text{d) } 7^{-\frac{3}{5}} \cdot 7^{-\frac{8}{5}}$$

$$7^{-\frac{3}{5}} \cdot 7^{-\frac{8}{5}} = 7^{-\frac{3}{5} - \frac{8}{5}} = 7^{-\frac{11}{5}} = \frac{1}{7^{\frac{11}{5}}} = \frac{1}{\sqrt[5]{7^{11}}}$$

$$\text{e) } \frac{5^{\frac{3}{4}}}{3} \cdot \frac{5^{-\frac{1}{4}}}{3}$$

$$\frac{5^{\frac{3}{4}}}{3} \cdot \frac{5^{-\frac{1}{4}}}{3} = \frac{5^{\frac{3}{4} - \frac{1}{4}}}{3} = \frac{5^{\frac{2}{4}}}{3} = \frac{5^{\frac{1}{2}}}{3} = \sqrt{\frac{5}{3}}$$

$$\text{f) } \frac{1^{-\frac{2}{3}}}{2} \cdot \frac{1^{-\frac{1}{2}}}{2}$$

$$\frac{1^{-\frac{2}{3}}}{2} \cdot \frac{1^{-\frac{1}{2}}}{2} = \frac{1^{-\frac{2}{3} - \frac{1}{2}}}{2} = \frac{1^{-\frac{4}{6} - \frac{3}{6}}}{2} = \frac{1^{-\frac{7}{6}}}{2} = \frac{1}{2^{\frac{7}{6}}} = 2^{-\frac{7}{6}} = \sqrt[6]{2^{-7}}$$