Aufgaben zu Ableitungen von Funktionen

1. Es sind k, m, n $\in \mathbb{Z}$. Bestimme jeweils die 1. Ableitung der Funktion.

a) $f(x) = x^k$

b) $f(x) = x^{5n}$

c) $f(x) = x^{4k+2}$

d) $f(x) = x^{3-4n}$

e) $f(x) = \frac{1}{x^{5-3k}}$

f) $f(x) = \frac{1}{x^{-2mn}}$

g) $f(x) = \frac{1}{x^{-(k-4)}}$

h) $f(x) = \frac{1}{x^{3\cdot(3-k)}}$

2. Leite jede Funktion 3-mal ab.

a) $f(x) = x^5 + 4x^4 - 3x^3$

b) $f(x) = -4x^6 + 2x^5 - x^2$

c) $f(x) = x^3 + 1{,}2x^2 - 10x$

d) $f(x) = \frac{5}{7}x^9 - 5x^6 + \frac{7}{3}x^5$

e) $f(x) = \frac{1}{10}x^5 + \frac{4}{9}x^3 + 8$

f) $f(x)=\frac{1}{3}x^6+\frac{6}{7}x^4-\frac{2}{5}x^3$

3. Leite mittels der Produktregel ab.

a) $f(x)=\sqrt{x}\cdot x$

b) $f(x)=x^2\cdot\sqrt{x}$

c) $f(k)=\sqrt{k}\cdot(3k+2)$

d) $f(t)=\sqrt{t}\cdot(2t^2-3)$

e) $f(t)=(1-2t^3)\cdot\sqrt{t}$

f) $g(a)=3a^4\cdot 4\sqrt{a}$

4. Wende die Quotientenregel für die 1. Ableitung an.

a) $f(x)=\frac{x}{2x+3}$

b) $f(x)=\frac{-x}{x^2+4}$

c) $f(x)=\frac{3-x}{3+x}$

d) $f(x)=\frac{1-x^2}{x+2}$

e) $f(x)=\frac{2x}{1+3x}$

f) $f(x)=\frac{3x^2-1}{x^2+4}$

Lösungen

1. Es sind k, m, n $\in \mathbb{Z}$. Bestimme jeweils die 1. Ableitung der Funktion.

a) $f(x) = x^k$

$f(x) = x^k$

$f'(x) = k \cdot x^{k-1} = kx^{k-1}$

b) $f(x) = x^{5n}$

$f(x) = x^{5n}$

$f'(x) = 5n \cdot x^{5n-1} = 5nx^{5n-1}$

c) $f(x) = x^{4k+2}$

$f(x) = x^{4k+2}$

$f'(x) = (4k+2) \cdot x^{4k+2-1} = (4k+2) \cdot x^{4k+1}$

d) $f(x) = x^{3-4n}$

$f(x) = x^{3-4n}$

$f'(x) = (3-4n) \cdot x^{3-4n-1} = (3-4n) \cdot x^{2-4n}$

e) $f(x)=\frac{1}{x^{5-3k}}$

$f(x)=\frac{1}{x^{5-3k}}=x^{-(5-3k)}=x^{-5+3k}$

$f'(x)=x^{-5+3k}=(-5+3k)\cdot x^{-5+3k-1}=(-5+3k)\cdot x^{-6+3k}=(3k-5)\cdot x^{3k-6}$

f) $f(x)=\frac{1}{x^{-2mn}}$

$f(x)=\frac{1}{x^{-2mn}}=x^{-(-2mn)}=x^{2mn}$

$f'(x)=(2mn)\cdot x^{2mn-1}$

g) $f(x)=\frac{1}{x^{-(k-4)}}$

$f(x)=\frac{1}{x^{-(k-4)}}=\frac{1}{x^{-k+4}}=x^{-(-k+4)}=x^{k-4}$

$f'(x)=(k-4)\cdot x^{k-4-1}=(k-4)\cdot x^{k-5}$

h) $f(x)=\frac{1}{x^{3\cdot(3-k)}}$

$f(x)=\frac{1}{x^{3\cdot(3-k)}}=\frac{1}{x^{9-3k}}=x^{-(9-3k)}=x^{-9+3k}$

$f'(x)=(-9+3k)\cdot x^{-9+3k-1}=(-9+3k)\cdot x^{-10+3k}=(3k-9)x^{3k-10}$

2. Leite jede Funktion 3-mal ab.

a) $f(x) = x^5 + 4x^4 - 3x^3$

$f(x) = x^5 + 4x^4 - 3x^3$

$f'(x) = 5 \cdot x^{5-1} + 4 \cdot 4x^{4-1} - 3 \cdot 3x^{3-1} = 5x^4 + 16x^3 - 9x^2$

$f''(x) = 4 \cdot 5x^{4-1} + 3 \cdot 16x^{3-1} - 2 \cdot 9x^{2-1} = 20x^3 + 48x^2 - 18x$

$f'''(x) = 3 \cdot 20x^{3-1} + 2 \cdot 48x^{2-1} - 1 \cdot 18x^{1-1} = 60x^2 + 96x - 18$

b) $f(x) = -4x^6 + 2x^5 - x^2$

$f(x) = -4x^6 + 2x^5 - x^2$

$f'(x) = 6 \cdot (-4)x^{6-1} + 5 \cdot 2x^{5-1} - 2 \cdot x^{2-1} = -24x^5 + 10x^4 - 2x$

$f''(x) = 5 \cdot (-24)x^{5-1} + 4 \cdot 10x^{4-1} - 1 \cdot 2x^{1-1} = -120x^4 + 40x^3 - 2$

$f'''(x) = 4 \cdot (-120)x^{4-1} + 3 \cdot 40x^{3-1} = -480x^3 + 120\, x^2$

c) $f(x) = x^3 + 1{,}2x^2 - 10x$

$f(x) = x^3 + 1{,}2x^2 - 10x$

$f'(x) = 3 \cdot x^{3-1} + 2 \cdot 1{,}2x^{2-1} + 1 \cdot (-10)x^{1-1} = 3x^2 + 2{,}4x - 10$

$f''(x) = 2 \cdot 3x^{2-1} + 1 \cdot 2{,}4x^{1-1} = 6x + 2{,}4$

$f'''(x) = 1 \cdot 6x^{1-1} = 6$

d) $f(x)=\frac{5}{7}x^9-5x^6+\frac{7}{3}x^5$

$f(x)=\frac{5}{7}x^9-5x^6+\frac{7}{3}x^5$

$f'(x)=9\cdot\frac{5}{7}x^{9-1}+6\cdot(-5)x^{6-1}+5\cdot\frac{7}{3}x^{5-1}=\frac{45}{7}x^8-30x^5+\frac{35}{3}x^4$

$f''(x)=8\cdot\frac{45}{7}x^{8-1}+5\cdot(-30)x^{5-1}+4\cdot\frac{35}{3}x^{4-1}=\frac{360}{7}x^7-150x^4+\frac{140}{3}x^3$

$f'''(x)=7\cdot\frac{360}{7}x^{7-1}+4\cdot(-150)x^{4-1}+3\cdot\frac{140}{3}x^{3-1}=360x^6-600x^3+140x^2$

e) $f(x)=\frac{1}{10}x^5+\frac{4}{9}x^3+8$

$f'(x)=5\cdot\frac{1}{10}x^{5-1}+3\cdot\frac{4}{9}x^{3-1}=\frac{1}{2}x^4+\frac{4}{3}x^2$

$f''(x)=4\cdot\frac{1}{2}x^{4-1}+2\cdot\frac{4}{3}x^{2-1}=2x^3+\frac{8}{3}x$

$f'''(x)=3\cdot 2x^{3-1}+1\cdot\frac{8}{3}x^{1-1}=6x^2+\frac{8}{3}$

f) $f(x)=\frac{1}{3}x^6+\frac{6}{7}x^4-\frac{2}{5}x^3$

$f(x)=\frac{1}{3}x^6+\frac{6}{7}x^4-\frac{2}{5}x^3$

$f'(x)=6\cdot\frac{1}{3}x^{6-1}+4\cdot\frac{6}{7}x^{4-1}+3\cdot(-\frac{2}{5})x^{3-1}=2x^5+\frac{24}{7}x^3-\frac{6}{5}x^2$

$f''(x)=5\cdot 2x^{5-1}+3\cdot\frac{24}{7}x^{3-1}+2\cdot(-\frac{6}{5})x^{2-1}=10x^4+\frac{72}{7}x^2-\frac{12}{5}x$

$f'''(x)=4\cdot 10x^{4-1}+2\cdot\frac{72}{7}x^{2-1}+1\cdot(-\frac{12}{5})x^{1-1}=40x^3+\frac{144}{7}x-\frac{12}{5}$

3. Leite mittels der Produktregel ab.

a) $f(x)=\sqrt{x}\cdot x$

$f(x)=\sqrt{x}\cdot x=x^{\frac{1}{2}}\cdot x$

$f'(x)=\frac{1}{2}\cdot x^{\frac{1}{2}-1}\cdot x+x^{\frac{1}{2}}\cdot x^{1-1}=\frac{1}{2}\cdot x^{-\frac{1}{2}}\cdot x+x^{\frac{1}{2}}\cdot x^0=\frac{1}{2}\cdot x^{-\frac{1}{2}+1}+x^{\frac{1}{2}}=\frac{1}{2}\cdot x^{\frac{1}{2}}+x^{\frac{1}{2}}=\frac{3}{2}x^{\frac{1}{2}}=\frac{3}{2}\sqrt{x}$

b) $f(x)=x^2\cdot\sqrt{x}$

$f(x)=x^2\cdot\sqrt{x}=x^2\cdot x^{\frac{1}{2}}$

$f'(x)=2\cdot x^{2-1}\cdot x^{\frac{1}{2}}+x^2\cdot\frac{1}{2}x^{\frac{1}{2}-1}=2x\cdot x^{\frac{1}{2}}+\frac{1}{2}x^2x^{-\frac{1}{2}}=2x^{1+\frac{1}{2}}+\frac{1}{2}x^{2-\frac{1}{2}}+2x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{3}{2}}=\frac{5}{2}x^{\frac{3}{2}}=\frac{5}{2}\sqrt{x^3}$

c) $f(k)=\sqrt{k}\cdot(3k+2)$

$f(k)=\sqrt{k}\cdot(3k+2)=k^{\frac{1}{2}}\cdot(3k+2)$

$f'(k)=\frac{1}{2}\cdot k^{\frac{1}{2}-1}\cdot(3k+2)+k^{\frac{1}{2}}\cdot 3=(3k+2)\cdot\frac{1}{2}\cdot k^{-\frac{1}{2}}+3k^{\frac{1}{2}}=\frac{3k+2}{2\sqrt{k}}+3\sqrt{k}=\frac{3k+2+3\sqrt{k}\cdot 2\sqrt{k}}{2\sqrt{k}}=\frac{9k+2}{2\sqrt{k}}=\frac{4{,}5k+1}{\sqrt{k}}$

d) $f(t)=\sqrt{t}\cdot(2t^2-3)$

$f(t)=\sqrt{t}\cdot(2t^2-3)=t^{\frac{1}{2}}\cdot(2t^2-3)$

$f'(t)=\frac{1}{2}\cdot t^{\frac{1}{2}-1}\cdot(2t^2-3)+t^{\frac{1}{2}}\cdot 2\cdot 2t^{2-1}=\frac{1}{2}\cdot t^{-\frac{1}{2}}\cdot(2t^2-3)+4t^{\frac{3}{2}}=t^{\frac{3}{2}}-\frac{3}{2}t^{-\frac{1}{2}}+4t^{\frac{3}{2}}=5t^{\frac{3}{2}}-\frac{3}{2}t^{-\frac{1}{2}}=5\sqrt{t^3}-\frac{3}{2\sqrt{t}}=\frac{10t^2-3}{2\sqrt{t}}$

e) $f(t)=(1-2t^3)\cdot\sqrt{t}$

$f(t)=(1-2t^3)\cdot\sqrt{t}=(1-2t^3)\cdot t^{\frac{1}{2}}$

$f(t)'=3\cdot(-2)t^{3-1}\cdot t^{\frac{1}{2}}+(1-2t^3)\cdot\frac{1}{2}\cdot t^{\frac{1}{2}-1}=-6t^2t^{\frac{1}{2}}+\frac{1}{2}(1-2t^3)t^{-\frac{1}{2}}=-6t^{\frac{5}{2}}+\frac{1}{2\sqrt{t}}-t^{\frac{5}{2}}=-7t^{\frac{5}{2}}+\frac{1}{2\sqrt{t}}=\frac{-14t^3+1}{2\sqrt{t}}$

f) $g(a)=3a^4\cdot 4\sqrt{a}$

$g(a)=3a^4\cdot 4\sqrt{a}=3a^4\cdot 4a^{\frac{1}{2}}$

$g'(a)=4\cdot 3a^{4-1}\cdot 4a^{\frac{1}{2}}+3a^4\cdot\frac{1}{2}\cdot 4a^{\frac{1}{2}-1}=48a^3\cdot a^{\frac{1}{2}}+6a^4\cdot a^{-\frac{1}{2}}=48a^{\frac{7}{2}}+6a^{\frac{7}{2}}=54a^{\frac{7}{2}}=54a^{3+\frac{1}{2}}=54a^3a^{\frac{1}{2}}=54a^3\sqrt{a}$

4. Wende die Quotientenregel für die 1. Ableitung an.

a) $f(x)=\frac{x}{2x+3}$

$f'(x)=\frac{1\cdot(2x+3)-x\cdot 2}{(2x+3)^2}=\frac{2x+3-2x}{(2x+3)^2}=\frac{3}{(2x+3)^2}$

b) $f(x)=\frac{-x}{x^2+4}$

$f'(x)=\frac{(-1)\cdot(x^2+4)-(-x)\cdot 2x}{(x^2+4)^2}=\frac{-x^2-4-(-2x^2)}{(x^2+4)^2}=\frac{x^2-4}{(x^2+4)^2}$

c) $f(x)=\frac{3-x}{3+x}$

$$f'(x)=\frac{(-1)\cdot(3+x)-(3-x\cdot 1)}{(3+x)^2}=\frac{-3-x-3+x}{(3+x)^2}=\frac{-6}{(3+x)^2}$$

d) $f(x)=\frac{1-x^2}{x+2}$

$$f'(x)=\frac{-2x\cdot(x+2)-(1-x^2)\cdot 1}{(x+2)^2}=\frac{-2x^2-4x-1+x^2}{(x+2)^2}=\frac{-x^2-4x-1}{(x+2)^2}$$

e) $f(x)=\frac{2x}{1+3x}$

$$f'(x)=\frac{2\cdot(1+3x)-2x\cdot 3}{(1+3x)^2}=\frac{2+6x-6x}{(1+3x)^2}=\frac{2}{(1+3x)^2}$$

f) $f(x)=\frac{3x^2-1}{x^2+4}$

$$f'(x)=\frac{6x\cdot(x^2+4)-(3x^2-1)\cdot 2x}{(x^2+4)^2}=\frac{6x^3+24x-6x^3+2x}{(x^2+4)^2}=\frac{26x}{(x^2+4)^2}$$